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Engg. Math.

- Gate
  - 13 M / 100
- ESE
  - Pre. I
    - 15 Q
    - 30 Marks / 200
  - Pre II
    - 15 Q
    - 30M / 300
  - mains

- Syllabus :

Gate

1. Linear algebra
2. Differential Equations
3. Vector Calculus
4. Diff. Equ^n
5. Complex Analysis
6. fourier series
7. Probability

ESE

1. Numerical methods

Linear Algebra

- Linear system & Linear transformation

$AX = B$ $AX = y$

Let, x = Cost of ~~q~~ pens

y = cost of pencil

	Pen	pencil	cost
I-day	3	2	50
II day	2	1	30

Linear eq^n :

$$3x + 2y = 50$$
$$2x + 1y = 30$$

by solving both, $x = 10$, $y = 10$

$$\underset{A}{\begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}} \underset{X}{\begin{bmatrix} x \\ y \end{bmatrix}} = \underset{B}{\begin{bmatrix} 50 \\ 30 \end{bmatrix}}$$ --- matrix Form

- Linear algebra deals with linear systems & Linear transformation.
- As every L.S. & L.T. can be expessed in matrix form & there by applying matrix methods.

Matrices:

def^n: The Arrangement of rows & Columns.

$$A = [a_{ij}]_{m \times n}$$

$m \times n$ — order of matrix

a_{ij} — element in i^{th} row & J^{th} Column's

m = no. of rows

n = no. of Column's

- If $m = n$, $A_{n \times n}$ is a square matrix.
- If $m \neq n$, $A_{m \times n}$ is a Rectangular matrix.
- Lower triangular matrix:

$$A = [a_{ij}]_{n \times n}$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$i<j$

$i>j$

Principle diagonal element

if above the principle diagonal elements are zero (0) then it is called as Lower triangular matrix.

$A = [a_{ij}]_{n \times n}$ is L.T.M.

if all $a_{ij} = 0$ for $i<j$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

- Upper triangular matrix:

if below the principle diagonal element are zero (0) then it is called as Upper triangular ~~element~~ matrix

$A = [a_{ij}]_{n \times n}$ is U.T.M.

if $a_{ij} = 0$ for $i>j$

$$U = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

- Diagonal matrix:

$D = [d_{ij}]_{n \times n}$ is a diagonal matrix,

if $d_{ij} = 0$ for $i \neq j$; e.g. $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

- Scalar matrix :

$A = [a_{ij}]_{n \times n}$ is a Scalar matrix, if all

$a_{ij} = 0$ for $i \neq j$

$a_{ii} = k$

e.g. $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

Operations on Matrices:

- Addition & Substraction:

$A \pm B$, if $o(A) = o(B)$

- Product of matrices:

If $A_{m \times n}$, $B_{p \times q}$

$(A \cdot B)_{m \times q}$ is possible only if $n = p$

(no. of columns of A = no. of rows of B)

Note:

$$\boxed{\begin{array}{l} ① \ AB \neq BA \\ ② \ A(BC) = (AB)C \end{array}}$$

◎ Formulae: Let, $A_{m \times n}$, $B_{p \times q}$ & $n = p$,

① The no. of scalar multiplications required to find

$$\boxed{(AB)_{m \times q} = m\,p\,q}$$

$a \underline{\times} B \cancel{+} \cancel{BC} + C \underline{\times} D$
into = scalar multiplication

② The no. of additions required to find $(AB)_{m \times q}$

$$\boxed{(AB)_{m \times q} = m(p-1)q}$$

Gate 2M

Q. Let $P_{4 \times 2}$, $Q_{2 \times 4}$, $R_{4 \times 1}$ be matrices find min^m. no. of multiplications required to find PQR?

Soln; we know that,

$(PQ)R = P(QR)$

Consider, $P(QR)$

$QR \rightarrow Q_{2 \times 4} \cdot R_{4 \times 1} \longrightarrow$ no. of multiplications $= 2.4.1 = 8$

$+$

$P_{4 \times 2}\,(QR)_{2 \times 1} \Longrightarrow$ no. of multiplications $= 4 \times 2 \times 1 = 8$

16

min^m. no. of mult^n $= 16$

Consider : $[(PQ)R]$

$P_{4 \times 2}\ Q_{2 \times 4} \Longrightarrow$ no. of mult^n $= 4 \times 2 \times 4 = 32$

$+$

$(PQ)_{4 \times 4} \cdot R_{4 \times 1} \Longrightarrow$ no. of mult^n $= 4 \times 4 \times 1 = 16$

48

max^m. no. of mult^n $= 48$

$$\text{min. no. of addition} = 2 \times 3 \times 1 = 6$$
$$+ \quad 4 \times 1 \times 1 = 4$$
$$\overline{\quad 10 \quad}$$

$$\text{max}^m \text{ no. of addition} = 4 \times 1 \times 4 = 16$$
$$+ \quad 4 \times 3 \times 1 = 12$$
$$\overline{\quad 28 \quad}$$

<u>Note</u>: The min. no. of multiplications required to get $PQR = 16$

max^m no. of multiplications required to find $PQR = 48$

The min^m no. of additions required $= 10$

max^m no. of additions required $= 28$

Transpose of Matrix :

If $A = [a_{ij}]_{m \times n}$, Then $A^T = [a_{ji}]_{n \times m}$

e.g. $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 6 & 7 \end{bmatrix}_{3 \times 2} \Longrightarrow A^T = \begin{bmatrix} 2 & 4 & 6 \\ 3 & 5 & 7 \end{bmatrix}_{2 \times 3}$