MADE EASY

Mechanical Engineering

Toppers Handwritten Notes

Fluid Mechanics

By Jitendra Gill Sir

PAROOL

Date ___ Page ___

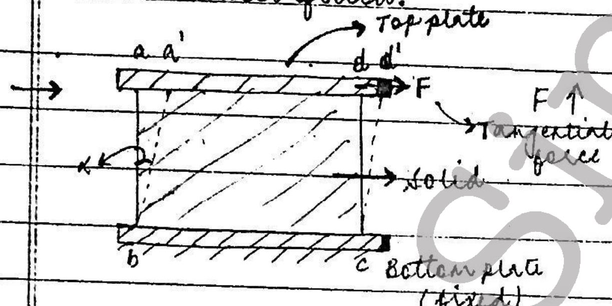
Fluid Mechanics

Fluid - Fluid can flow → continuous plastic deformation

deformation → elastic, plastic

Matter → solid, liquid, gases; liquid + gases ⇒ Fluid → Fluid Mechanics

Dye - coloured fluid which is immiscible and has same density (almost) as the test fluid.

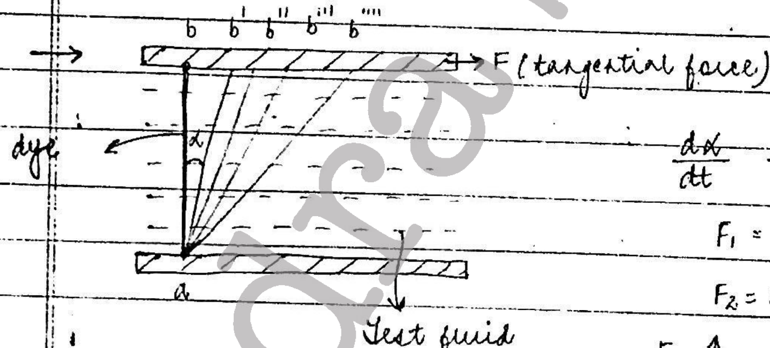


$F \uparrow \quad \alpha \uparrow$

$F_1 = 100\,N \quad \alpha_1 = 0.1°$

$F_2 = 1000\,N \quad \alpha_2 = 0.05°$

$F_3 = 2000\,N \quad \alpha_3 = 0.07°$



$\frac{d\alpha}{dt}$ - Rate of shear strain

$F_1 = 10\,N \quad \frac{d\alpha}{dt} = 0.15$

$F_2 = 100\,N \quad \frac{d\alpha}{dt} = 0.55$

$F - \uparrow \quad \frac{d\alpha}{dt}\uparrow$ for same fluid

const ⇒

→ **Fluid** is a substance that deforms continuously under the action of tangential force/shear stress no matter how small the tangential force is.

→ Fluid cannot resist shear stress under static conditions.

- In fluids, rate of deformation is more important than total deformation because fluid continues to deform as long as external shear forces are applied. (Fluids do not flow under normal force; they flow only under tangential force).

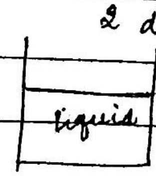
→ Characteristics of solids, liquids and gases

Characteristic	Solid	Liquid	gas
1. Response to shear stress	Resists total deformation	Resists rate of deformation	
2. Compressibility (β)	Incompressible	Virtually incompressible	Compressible
3. Ability to conform shape of container	No	Yes	yes
4. Ability to expand without limits	No (Cohesion - max)	No	yes
5. Ability to form free surface	yes	Yes	No
6. Ability to resist small amount of tensile stress	yes	Yes	No

Ability to resist small amount of tensile stress
↓
bond strength
solid >>> liq > gas.

Expand without limits
↓
cohesive forces
solid >> liquid > gas.

Free surface - interface b/w 2 different media.



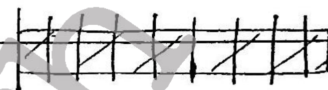
→ Free surface.

→ **Concept of continuum** → Assumption

↓

Continuous distribution of mass.

If continuum fails, we can't write fluid properties as a continuous function of space variables.

$T = T_0\left(1 - \frac{x}{L}\right)$ 

↓

Continuous function

Mass is distributed uniformly.

- Continuum fails in
  1) High order of vaccum.
  2) Rarefied gas flow → Gas with very low density
     eg:- flight of rocket in upper atmosphere.

→ **Fluid properties**.

1. **Density - (ρ)** ⇒

$$\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{M}{V} = \frac{kg}{M^3}$$

$cgs \Rightarrow g/cm^3$ or g/cc

$\therefore \; 1g/cc = 10^{-3}/10^{-6} \; kg/m^3 = 1000 \, kg/m^3$.

$\rho_{liq} >> \rho_{gas}$

↳ at same temperature & pressure

$P \uparrow \quad \rho \uparrow$

$T \uparrow \quad \rho \downarrow$

2. **Specific volume (v) ⇒**

$$v = \frac{\text{Volume}}{\text{mass}} = \frac{m^3}{kg}$$

$$v = \frac{1}{\rho}$$

3. **Specific gravity ⇒**

$$S.g. = \frac{\text{Density of fluid}}{\text{Density of standard fluid}}$$

Standard fluid

- Water (In case of liquid)
  $\rho_w = 998\ kg/m^3$
  at 1 atm & 20°C
- Air / H_2 (In case of gases)
  $\rho_{air} = 1.2\ kg/m^3$
  at 1 atm & 20°C.
  $\rho_{H_2} = 0.084\ kg/m^3$
  at 1 atm & 20°C.

4. **Specific weight (γ) ⇒**

$$\gamma = \frac{\text{Weight}}{\text{Volume}} = \frac{mg}{V} \left(\frac{N}{m^3}\right)$$

$$\left(\frac{m}{V}\right) g = \gamma$$

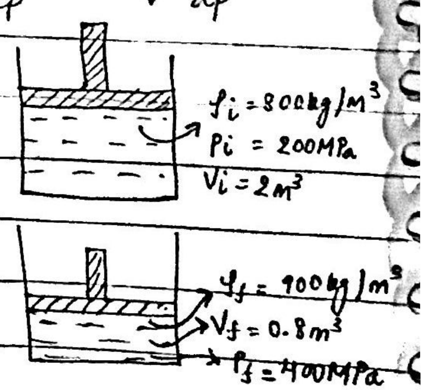
$$\boxed{\rho g = \gamma}$$

5. **Compressibility (β) ⇒**

$$\beta = -\frac{\text{Volumetric strain}}{\text{change in pressure}} = -\frac{\left(\frac{dV}{V}\right)}{dp} = -\frac{1}{V}\frac{dV}{dp}$$

$$\beta = -\frac{V_f - V_i}{V_i (P_f - P_i)}$$

β → avg compressibility



For closed system -

$$M = \text{constant } (c)$$

$$\rho V = c$$

$$\ln(\rho V) = \ln c$$

$$\ln \rho + \ln V = \ln c$$

$$\frac{d\rho}{\rho} + \frac{dV}{V} = 0 \Rightarrow \boxed{\frac{d\rho}{\rho} = -\frac{dV}{V}}$$

$$\Rightarrow \beta = \frac{-\left(\frac{dV}{V}\right)}{dp} = \frac{\frac{d\rho}{\rho}}{dp} = \frac{1}{\rho}\frac{d\rho}{dp}$$

$$\boxed{\beta = \frac{\rho_f - \rho_i}{\rho_i (P_f - P_i)}}$$

Perfectly incompressible fluid.

$$\boxed{\beta = 0}$$

$\rightarrow$ If $\beta_A = 2.1 \times 10^{-9}\ Pa^{-1}$ $\quad \beta_B = 2.6 \times 10^{-7}\ Pa^{-1}$

$$\boxed{\beta_A < \beta_B}$$

$\Rightarrow$ Higher the value of β, more compressible fluid will be.

βB is more compressible than A.

Compressibility
- $\rightarrow$ Isothermal compressibility (compression process is isothermal) $\Rightarrow T = C$
- $\rightarrow$ Adiabatic compressibility (Compression process is adiabatic) $\Downarrow$ No heat transfer

Ideal gas.

1. $P V = \underset{\substack{\downarrow \\ \text{No of moles}}}{n} \bar{R} T$ $\rightarrow$ universal gas constant

2. $P V = \left(\frac{M}{\bar{M}}\right) \bar{R} T$

$$\boxed{P V = M R T}$$

$R \downarrow$ characteristic gas constant

3. $P = \frac{M}{V} R T$

$$\boxed{P = \rho R T}$$

→ **Isothermal compressibility for Ideal gas.**

$$\beta_T = \frac{1}{\rho}\left(\frac{d\rho}{dP}\right)_T = \frac{1}{\rho RT} = \frac{1}{P}.$$

$\frac{P}{\rho} = c$ — For isothermal process.

$$P = \rho RT$$

$$dP = RT\,d\rho$$

$$\left(\frac{d\rho}{dP}\right)_T = \frac{1}{RT}$$

$$\boxed{\beta_T = \frac{1}{P}}$$

P → absolute pressure.

→ **Adiabatic compressibility for Ideal gas.**

$$\boxed{\frac{P}{\rho^\gamma} = c}$$ — For Adiabatic process

γ – ratio of specific heat of a gas at constant pressure & constant volume. 1.4 for air.

$$P = c\rho^\gamma$$

$$dP = c\gamma\rho^{\gamma-1}d\rho$$

$$\frac{d\rho}{dP} = \frac{1}{c\gamma\rho^{\gamma-1}}$$

$$PV^\gamma = C$$

$$\frac{PV^\gamma}{M^\gamma} = \frac{C}{M^\gamma}$$

$$\boxed{\frac{P}{\rho^\gamma} = C_1}$$

$$\boxed{P = C.\rho^\gamma}$$

$$\frac{1}{\rho}\left(\frac{d\rho}{dP}\right) = \frac{1}{(c\rho^\gamma)\gamma}$$

$$\boxed{\beta_{adia} = \frac{1}{\gamma P}}$$

→ $\beta_{iso} > \beta_{adia}$

i.e., isothermal compression is easier than adiabatic

→ Pressure ↑, compressibility becomes difficult.

6. Bulk modulus (K) ⇒

$$K = \frac{1}{\beta} = -V\frac{dP}{dV} = \rho\left(\frac{dP}{d\rho}\right) \quad Pa$$

$$K = -V_i\frac{(P_f - P_i)}{(V_f - V_i)} = \frac{\rho_i\,(P_f - P_i)}{(\rho_f - \rho_i)} \quad Pa$$

→ Perfectly incompressible fluid

$$K = \infty$$

$$K_A = 8 \times 10^9 \ Pa.$$

$$K_B = 7 \times 10^{11} \ Pa.$$

$$K_A < K_B$$

- Smaller the value of K, more compressible fluid will be.

A is more compressible than B.

→ Isothermal bulk modulus of ideal gas

$$K = \rho\left(\frac{dP}{d\rho}\right)_T = P$$

→ Adiabatic bulk modulus for ideal gas:

$$K = \rho\left(\frac{dP}{d\rho}\right)_{Adiabatic} = \gamma P$$

$$K_{adia} > K_{iso}$$

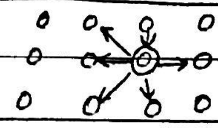
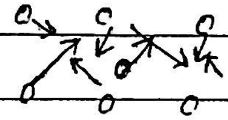
$$K_{liq} >> K_{gas}$$

$$\beta_{liq} << \beta_{gas}$$

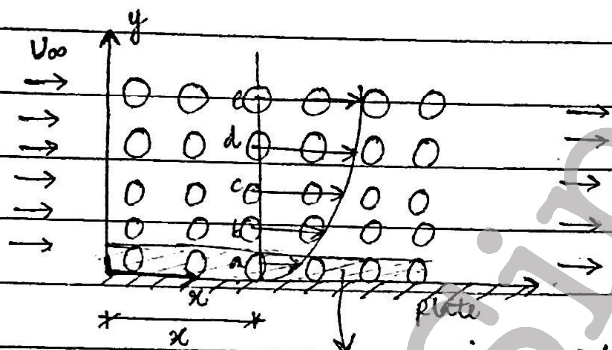
$$K_{water} >> K_{air}$$

$$\beta_{water} << \beta_{air}$$

7. Viscosity – Resistence to flow

Type of fluid	Reason
Liquid	 Molecular cohesive force.
gas	 Molecular momentum exchange (Random motion)

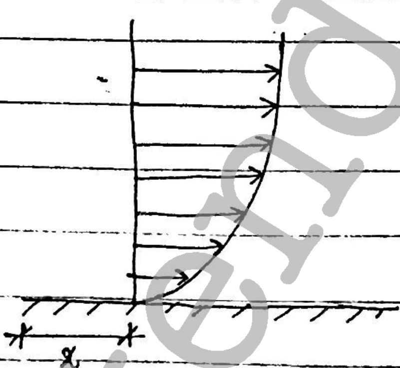
→ **No slip condition**

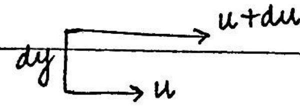


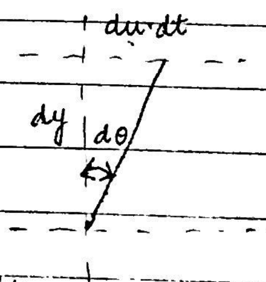
fluid in direct contact with surface will stick to the surface.

Velocity profile, $\boxed{u = u(y)}$

→ **Newton's law of viscosity**







$$\tan d\theta = \frac{du \cdot dt}{dy}$$

$d\theta$ – very small angle

$$\tan d\theta \simeq d\theta$$

$$d\theta = \frac{du \cdot dt}{dy}$$

$$\boxed{\frac{d\theta}{dt} = \frac{du}{dy}}$$

$\frac{du}{dy}$ → Velocity gradient in transverse dirn

$\frac{d\theta}{dt}$ → Rate of angular deformation or rate of shear strain

As per Newton's law of viscosity :-

$$\tau \propto \frac{d\theta}{dt} \quad \text{or} \quad \underset{\text{shear stress}}{\tau} \propto \dot{\theta} \rightarrow \text{Rate of shear strain}$$

$$\tau = \mu \frac{d\theta}{dt} = \mu \dot{\theta}$$

$$\boxed{\tau = \mu \frac{du}{dy}} \longrightarrow \text{Newton's law of viscosity}$$

I. $\boxed{\mu = \text{Dynamic viscosity}}$

$$\mu = \frac{\tau}{\left(\frac{du}{dy}\right)} = \frac{Pa}{\left(\frac{1}{s^{-1}}\right)} = Pa\text{-}s$$

SI unit $\Rightarrow$ Pa-s

MKS unit $\Rightarrow \frac{N}{m^2} \cdot s = \frac{kg\, m}{m^2 s^2} \times s = kg/ms$

CGS unit $\Rightarrow \frac{g}{cm\text{-}s} = $ Poise

1 Poise = 1 g/cms

$$\boxed{1\ \text{Poise} = \frac{10^{-3}\ kg}{10^{-2}\ ms} = 0.1\ kg/ms}$$

II. $\boxed{\nu = \text{Kinematic Viscosity}}$

$$\boxed{\nu = \frac{\mu}{\rho}} = \frac{kg/ms}{kg/m^3} = m^2/s$$

$m^2/s \rightarrow$ SI & MKS unit.

CGS unit $\Rightarrow cm^2/s$

$$\boxed{1\ \text{Stoke} = 1\ cm^2/s}$$

$$\boxed{1\ \text{Stoke} = 10^{-4}\ m^2/s}$$

- Parameters that do not have "mass" terms are called kinematic parameters. ν - Kinematic viscosity
- It represents momentum diffusability.

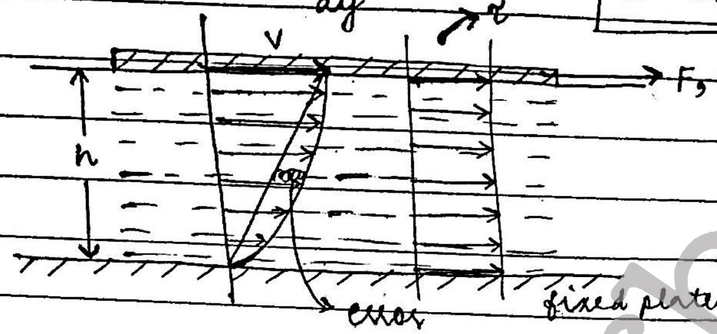
ν signifies distance upto which this effect is seen.

momentum of this layer killed

→ Linearization of Newton's law of viscosity.

$$\tau = \mu \frac{du}{dy}$$

$\boxed{u = u(y)}$ → Velocity profile



If h is very small, we can assume linear velocity profile.

$$u = a + by$$

a & b are arbitrary constants

Boundary conditions ⇒

1. $y = 0$, $u = 0$
2. $y = h$, $u = V$

$y = 0$, $u = 0$ — $\boxed{a = 0}$

$y = h$, $u = V$ $\quad V = bh$

$\Rightarrow \boxed{b = V/h}$

$\boxed{u = \frac{Vy}{h}}$ → linear velocity profile

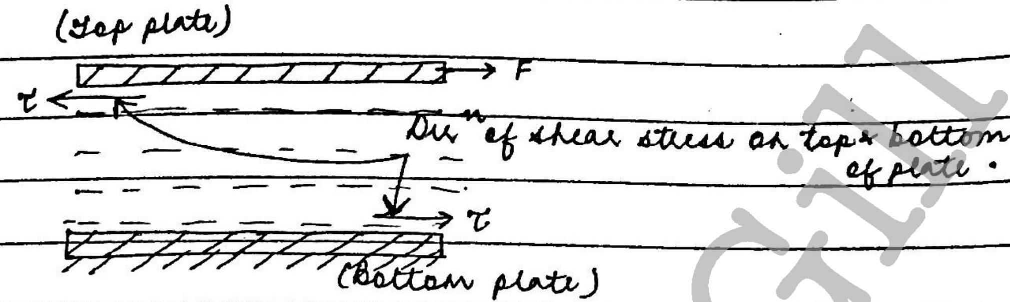
$$\tau = \mu \frac{du}{dy}$$

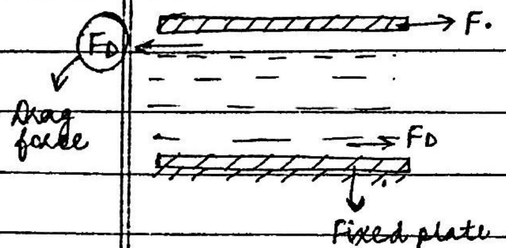
$$\tau = \mu \frac{d}{dy}\left(\frac{V \cdot y}{h}\right)$$

$$\boxed{\tau = \frac{\mu V}{h}}$$

$$\boxed{\begin{array}{l} \tau \propto \mu \\ \propto V \\ \propto 1/h \end{array}}$$

Shear stress is constant for linear velocity profile.





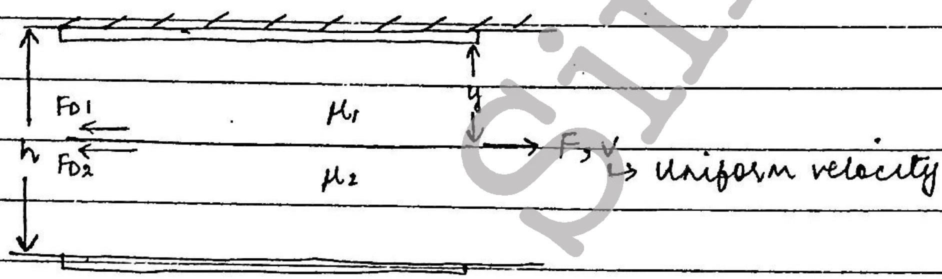
$F_D = \tau A$

↳ shear stress constant over entire area A.

$$\boxed{\tau_w = \frac{\mu V}{h}}$$

Workbook

(Q.34.)



Find 'y' such that F_D is minimum

$$\text{Total } F_D = F_{D1} + F_{D2}$$

$$= \frac{\mu_1 V}{y} A + \frac{\mu_2 V}{h-y} A$$

$$\frac{dF_D}{dy} = 0 \Rightarrow -\frac{\mu_1 VA}{y^2} + \frac{\mu_2 VA}{(h-y)^2} = 0$$

$$VA\left(-\frac{\mu_1}{y^2} + \frac{\mu_2}{(h-y)^2}\right) = 0$$

$$\frac{\mu_1}{y^2} = \frac{\mu_2}{(h-y)^2}$$

$$\frac{\sqrt{\mu_1}}{y} = \frac{\sqrt{\mu_2}}{(h-y)}$$

$$\sqrt{\mu_1}(h-y) = \sqrt{\mu_2} \cdot y$$

$$\boxed{\frac{\sqrt{\mu_1} \times h}{\sqrt{\mu_1} + \sqrt{\mu_2}} = y}$$

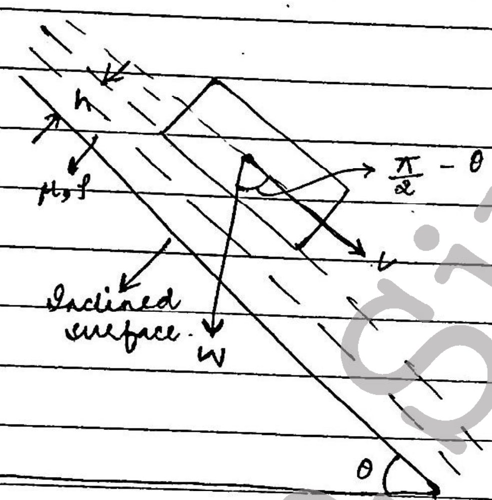
ii) Find the value of y for which F_D is equal at top & bottom of plate.

$$F_{D1} = F_{D2}$$

$$\frac{\mu_1 \cancel{V}}{y} A = \frac{\mu_2 \cancel{V} A}{(h-y)}$$

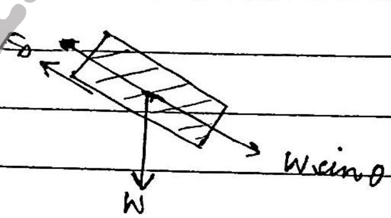
$$\mu_1 (h-y) = \mu_2 y$$

$$\therefore \boxed{\frac{\mu_1 h}{\mu_1 + \mu_2} = y}$$



Assume linear velocity profile.

V – Terminal velocity



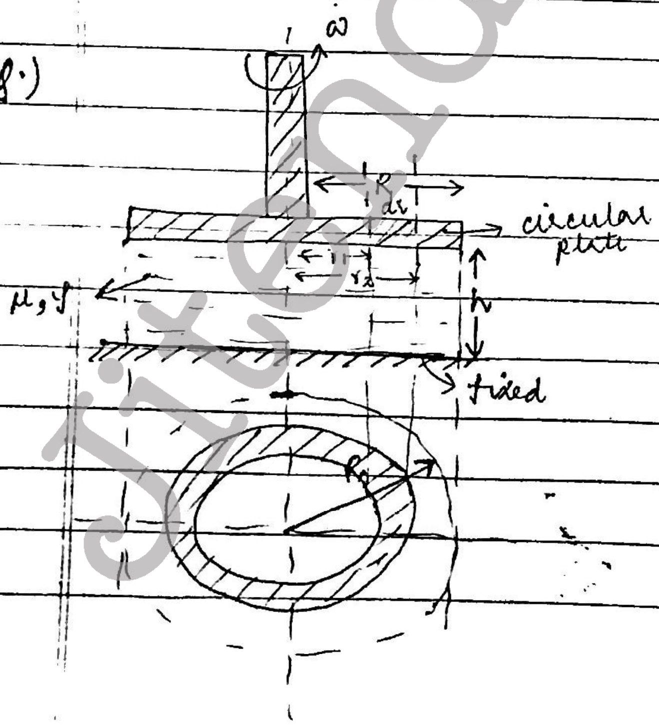
$$F_D \uparrow = \frac{\mu V}{h} \uparrow$$

$$F_D = W \sin\theta$$

$$\frac{\mu V}{h} A = W \sin\theta$$

$$\boxed{V = \frac{(W \sin\theta) h}{\mu A}}$$

Q.)



$F_D = ?$ $P = ?$

~~$F_D =$~~ V is not const
$\Rightarrow \tau$ is not const.

$$F_D = \int \tau \, dA$$

$$= \int \tau \times 2\pi r \, dr = \int_0^R \underbrace{\frac{\mu \omega r}{h} \times 2\pi r \, dr}_{dF_D}$$

Elemental drag force

$$= \frac{\mu \omega}{h} \times 2\pi \times \frac{r^3}{3}$$

$$\boxed{F_D = \frac{2\pi \mu \omega r^3}{3h}}$$