Basics of Laplace Transform - Part I

Comprehensive Course on Control System

Vishal Soni • Lesson 1 • Feb 9, 2022

Linear Control System

Syllabus

1. BASICS of SIGNAL
2. BLOCK DIAGRAM, SFG
3. TIME RESPONSE → Controller.
4. ROUTH HURWITZ CRITERIA
5. Root Locus
6. FREQUENCY RESPONSE
7. BODE PLOT
8. NYQUIST PLOT. → compensator.
9. STATE SPACE
10. MATHEMATICAL MODELLING
12. Properties / Missc.

Resources

→ Class Notes : PDF, Print out

→ QR + PR + DPP

→ PYQ: ECE, EE, IN → GATE

ECE, EE → ESE (LAST 10 year) (objective)

- Kanodiya ✓✓
- Bits- Bytes

- TEST SERIES → ✓✓ → Completion

Books

- B.C. KUO
- Norman Nise

Revision

- QR+PR
- 1 : ✓
  2 : QR+PR+DPP + 1 QR
  40 : QR+PR+DP+ (previ 39 QR.)

Short Notes: → Formulae ✓

Mon-Fri : Saturday: "8pm" →

Sunday: 12:15 pm →

Doubts

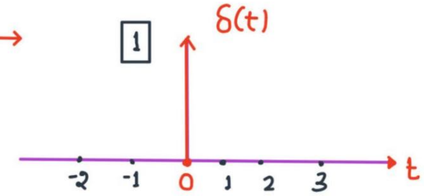
- → Doubt Solving Session
- → Feature
- → Doubt solving Batch

SIGNAL

(1) Unit Impulse Signal

→ $\boxed{x(t) = 1 \cdot \delta(t)}$

→ $\boxed{1}$



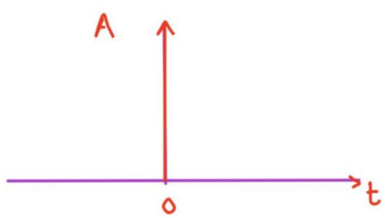
$\delta(t) = 0 \ : \ t \neq 0$

$\delta(t) \neq 0 \ : \ t = 0 \ : \ \delta(t) \to \infty$

$$\boxed{\int_{-\infty}^{\infty} \delta(t)\,dt = 1}$$

(2) Impulse Signal

$x(t) = A\,\delta(t)$

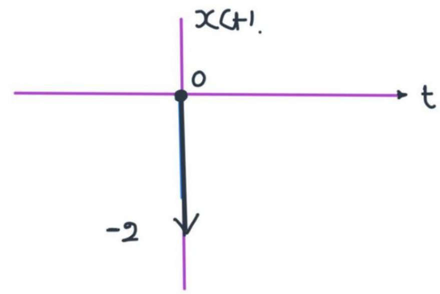


$A\delta(t) = 0 \quad t \neq 0$

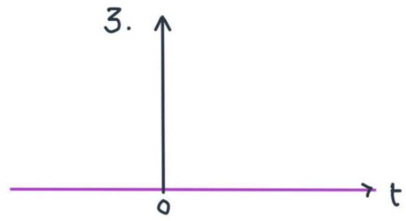
$A\delta(t) \neq 0 \quad t = 0 \ : \ A\delta(t) \to \infty$

$$\boxed{\int_{-\infty}^{\infty} A\delta(t)\,dt = A\int_{-\infty}^{\infty} \delta(t)\,dt = A}$$

1.) $x(t) = -2\,\delta(t)$



2.) $3\delta(t)$

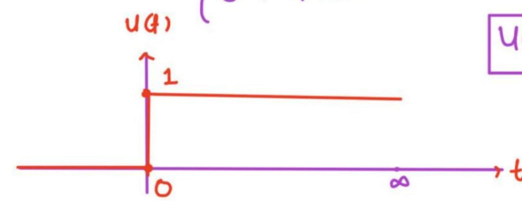


Note: "Impulse Signals have constant area."

(3) Unit Step Signal

→ $x(t) = u(t)$

→ $u(t) = \begin{cases} 1 : & t \geq 0 \\ 0 : & t < 0 \end{cases}$



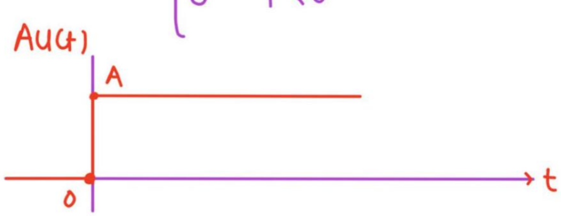
$\boxed{u(\infty) = 1}$

(4) Step Signal

STEP signals have constant amplitude

$x(t) = Au(t)$

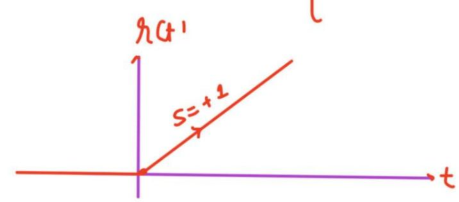
$x(t) = \begin{cases} A & t \geq 0 \\ 0 & t < 0 \end{cases}$



(5) Unit Ramp Signal

→ $x(t) = r(t)$

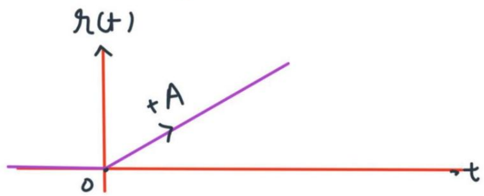
→ $r(t) = t\,\underline{u(t)} = \begin{cases} 0 : & t < 0 \\ t : & t \geq 0 \end{cases}$



(6) Ramp Signal

$x(t) = A\,r(t)$

$$A\,r(t) = A\,t\,u(t) = \begin{cases} At : & t \geq 0 \\ 0 : & t < 0 \end{cases}$$



NOTE: In Ramp Signals slope is always constant.

AREA vs INTEGRATION

1) $\boxed{\int_{-\infty}^{\infty} x(t)\,dt = \text{Area of } x(t)}$

2.) $\boxed{\int_{-\infty}^{t} x(t)\,dt = \text{Running Intergration of } x(t)}$

RELATION BETWEEN δ(t), u(t) and tu(t)

1) $\boxed{\int_{-\infty}^{t} \delta(t)\,dt = u(t)}$

3.) $\boxed{\frac{dr(t)}{dt} = u(t)}$

2.) $\boxed{\int_{-\infty}^{t} u(t)\,dt = r(t)}$

4.) $\boxed{\frac{du(t)}{dt} = \delta(t)}$

(7) Parabolic Signal

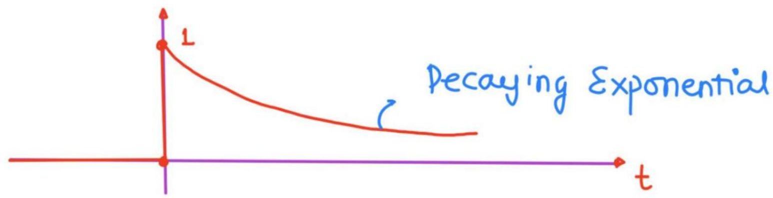
$$\boxed{p(t) = \frac{A}{2} t^2 u(t)}$$

Q. $p(t) = K t^2 u(t)$ Value of K for it to be a valid parabolic signal

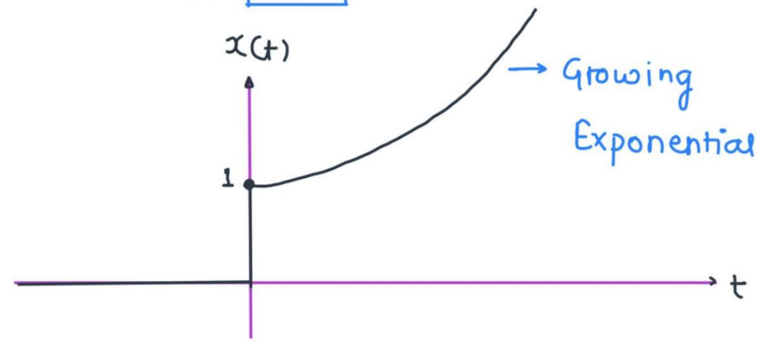
$\frac{2K}{2} t^2 u(t)$

$\boxed{A = 2K}$ $\boxed{K = A/2}$

(8) **One sided exponential** $e^{-at} u(t) : a > 0$



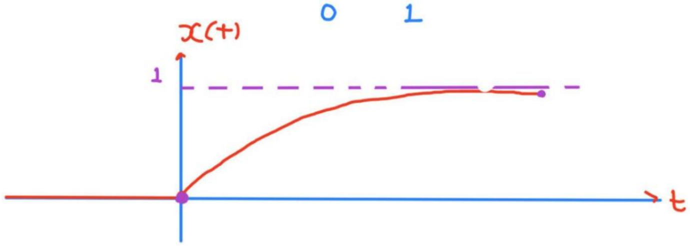
(9) $x(t) = e^{at} \underbrace{u(t)}$ $a > 0$



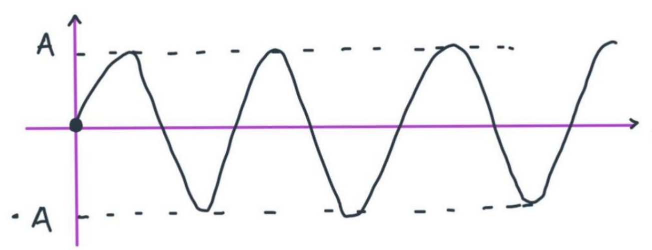
(10) $x(t) = \left(1 - e^{-at}\right) u(t)$ $a > 0$

$x(0) = (1 - e^{0})\, u(0) = 0$

$x(\infty) = (1 - \underset{\downarrow\, 0}{e^{-\infty}})\, \underset{\downarrow\, 1}{u(\infty)} = 1$

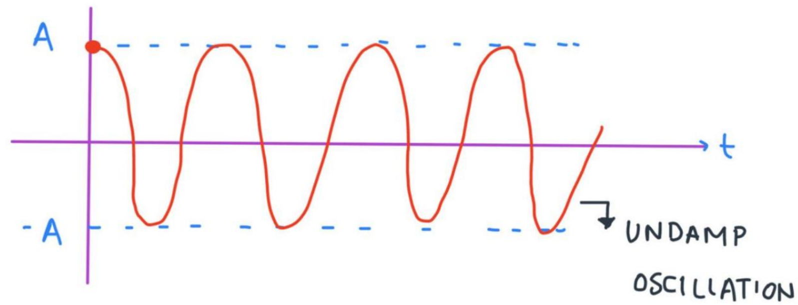


(11) $A \sin w_o t u(t) = \underbrace{\{A u(t)\}} \sin w_o t \xrightarrow{t=0} A \times 0 = 0$



(12)

$A\cos w_o t u(t) = \underbrace{\{A u(t)\}}\cos w_o t \xrightarrow{t=0} A \times \cos 0 = A$



Procedure to plot : $x(t) = \underbrace{A(t)}\,\underbrace{\cos w_o t}$ **or** $\underbrace{A(t)}\,\underbrace{\sin w_o t}$

Step 1: $\boxed{\text{Plot A(t) in Ref form (dotted)}}$

Step 2 : $\boxed{\text{Take mirror image of A(t) w.r.t. Horizontal axis.}}$

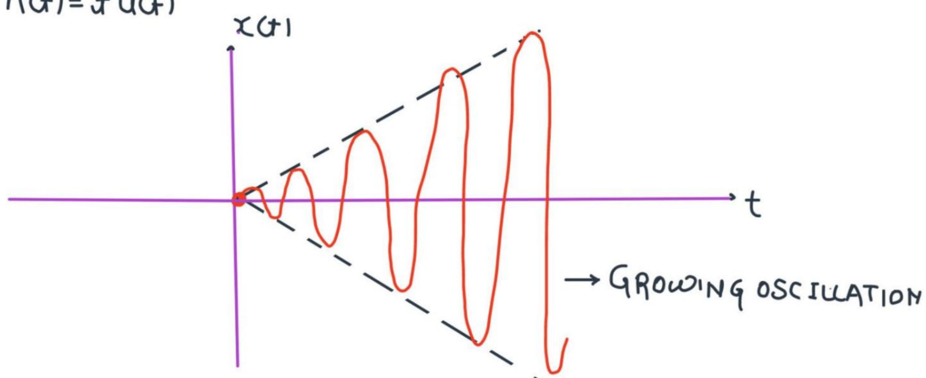
Step 3 : $\boxed{\text{Calculate x(t) at t = 0}}$

Step 4: $\boxed{\text{Draw sinusoidal shape in between}}$ ✓

Q-1

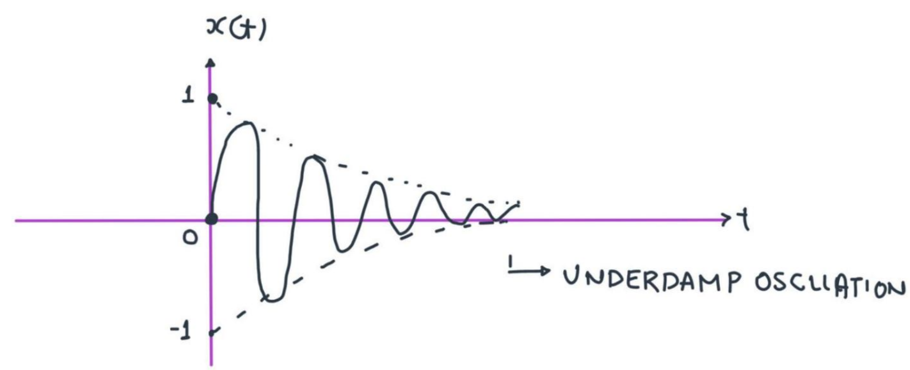
$x(t) = t\sin t\, u(t) = \{t u(t)\}\sin t \xrightarrow{t=0} 0$

$A(t) = t u(t)$

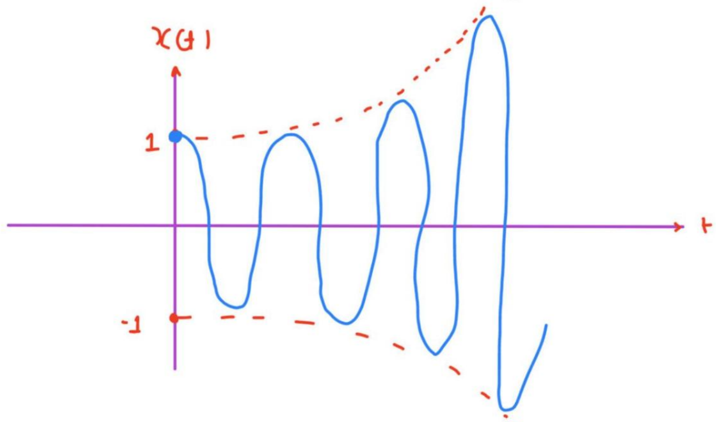


(13). 2

$x(t) = e^{-t}\sin t\, u(t) = \underbrace{(e^{-t} u(t))}_{A(t)}\sin t \xrightarrow{t=0} 0.$



(14) $x(t) = e^{2t} \cos t\, u(t) = \{e^{2t} u(t)\} \cos t \xrightarrow{t=0} 1$



BASIC OPERATION

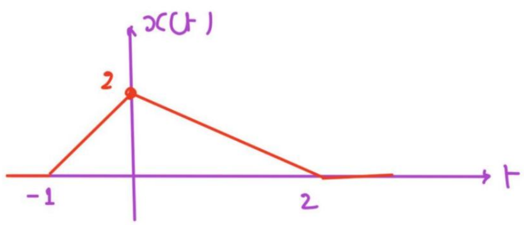
1.) Amplitude scaling

Given: $x(t)$ vs t

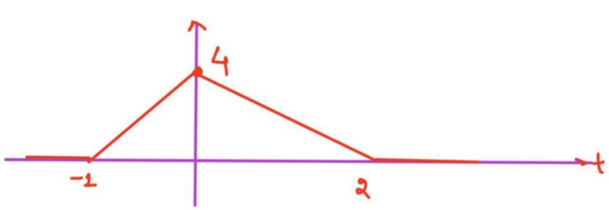
Plot: $A\,x(t)$ vs t

$\boxed{\text{method}}$: multiply vertical axis by A.

Q.



$y(t) = 2x(t)$

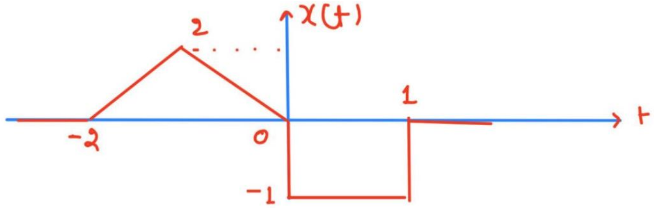


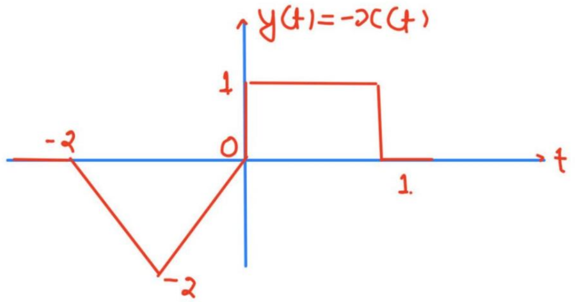
2.) Amplitude Reversal:

Given $x(t)$ vs t

Plot: $-x(t)$ vs t

$\boxed{\text{method:}}$ Mirror image w.r.t. Horizontal axis.





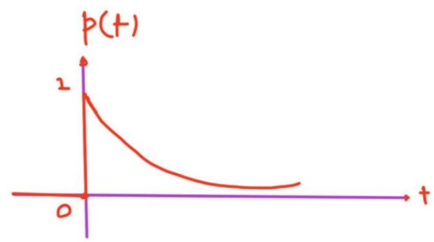
3.) Time shifting:

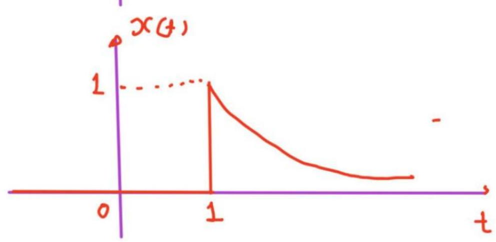
Given: $x(t)$ vs t

Plot: $\begin{cases} x(t-t_0) \text{ vs } t \longrightarrow \text{Right shift} \\ x(t+t_0) \text{ vs } t \longrightarrow \text{Left shift} \end{cases}$

Q.) $x(t) = e^{-2(t-1)}u(t-1)$

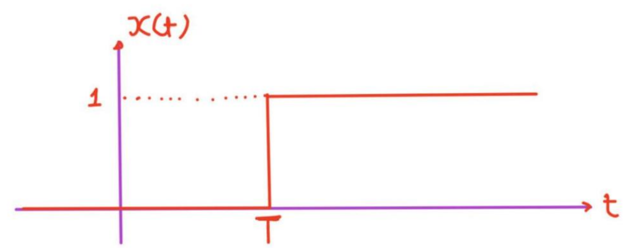
$$\boxed{p(t) = e^{-2t}u(t)}$$





Que.

$x(t) = u(t-T)$



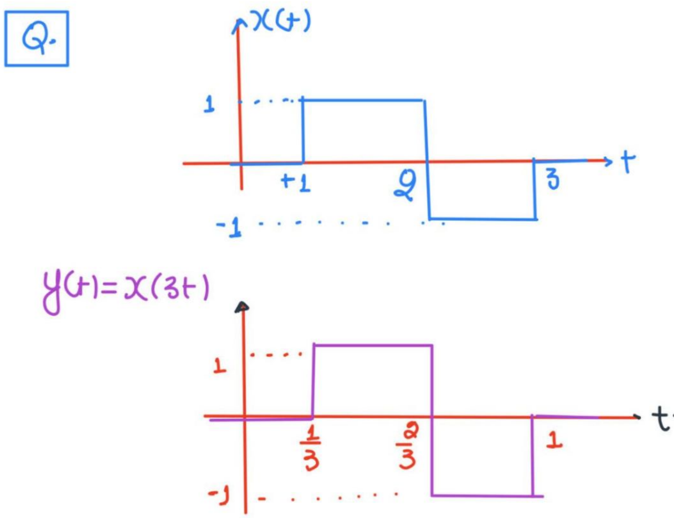
4.) <u>Time Scaling</u>:

Given $x(t)$ vs t

Plot $x(at)$ vs t

Method: divide horizontal axis by a.

Q.



5. <u>Time Reversal:</u>

Given: $x(t)$ vs t

Plot: $x(-t)$ vs t

Method: Mirror image w.r.t. vertical axis.