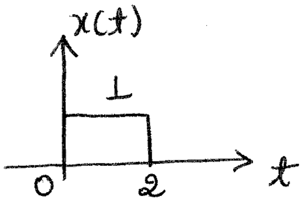
# -: SIGNAL SYSTEM :-

## Different operations on signal :

### 1. Time shifting :

(a) left shifting (b) Right shifting




$\longrightarrow x(t+K)$

where K is a real constant.

**case-(i) :** when $K>0 \rightarrow$ left shifting

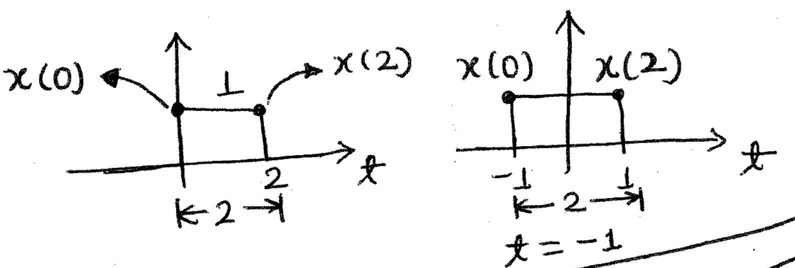
Ex: $K = 1$

$x(t) \longrightarrow x(t+1)$

$t \longrightarrow t+1$

$x(t+1) \longrightarrow x(0)$

$x(t+1) \longrightarrow x(2)$




$t=-1$

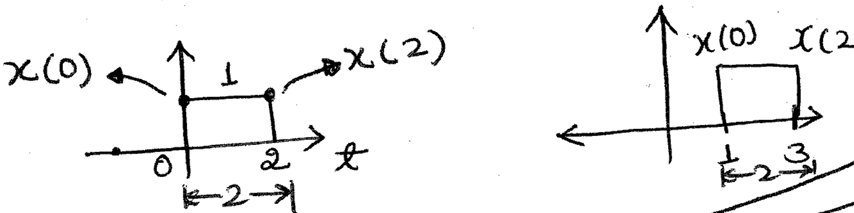
$x(0)$ $\quad t=1$

$x(2)$

**case-ii :** when $K<0 \rightarrow$ Right-shifting

Ex: $K = -1$

$x(t) \longrightarrow x(t-1)$




$t=1$

$x(0)$

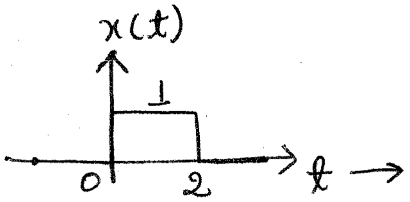
$x(2)$ $\quad t=3$

observation :- Extension get changed but time duration remains constant / Amplitude also.

### 2. Time - scaling :

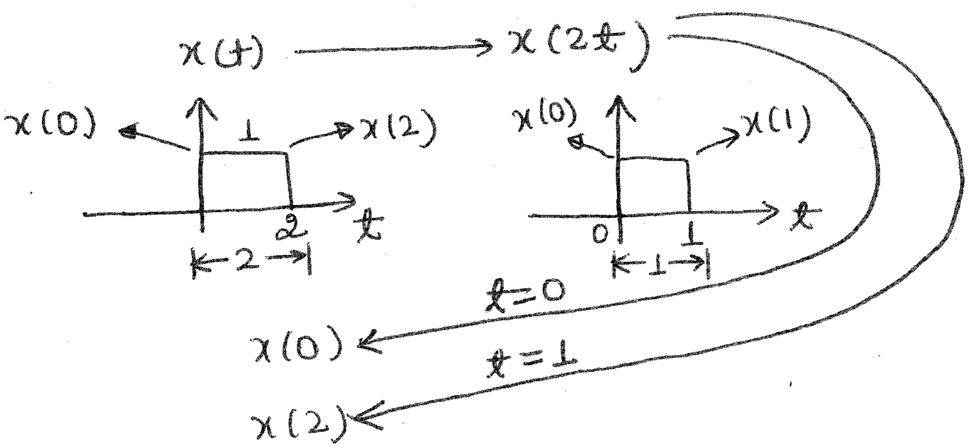
$x(t) \longrightarrow x(at)$, $a \neq 0$

and 'a' is real constant



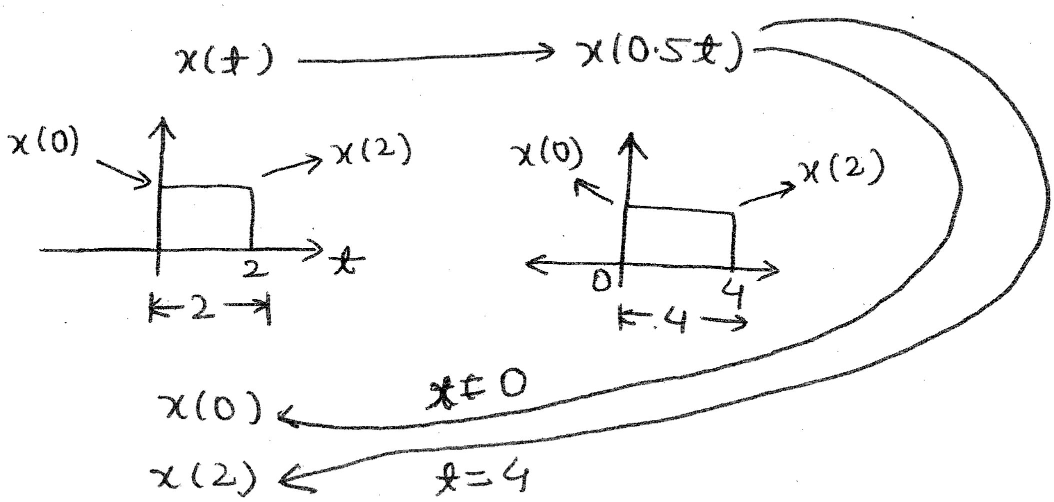
**Case (i) : when $a > 1 \rightarrow$ Time compression**

Ex : $a = 2$

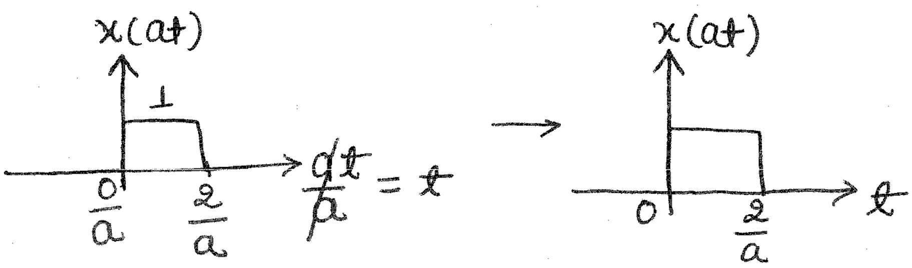



**Case (ii) : when $0 < a < 1 \rightarrow$ Time expansion**

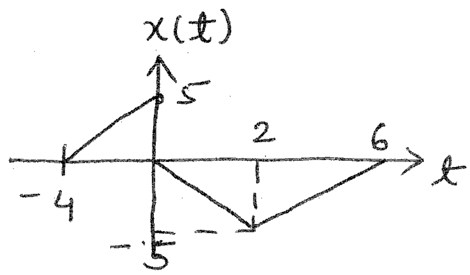
Ex : $a = 0.5$




**General rule :** $x(t) \longrightarrow x(at)$ w.r.t variable $t$ by default



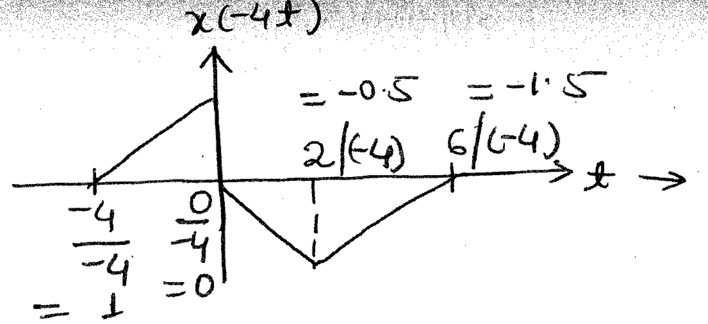

Ex- :

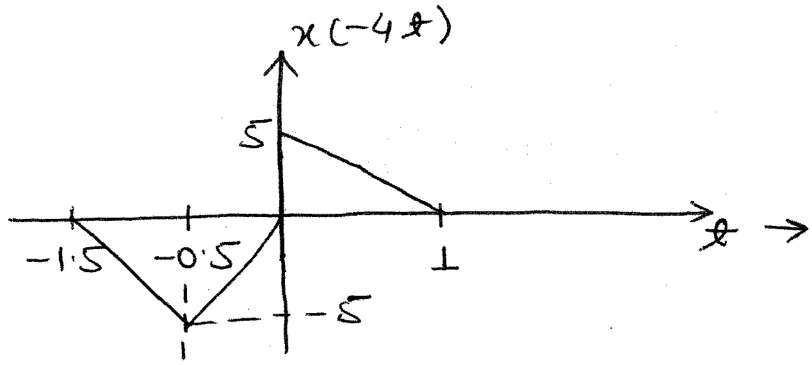



Draw waveform of signal

$y(t) = x(-4t)$

solution :-

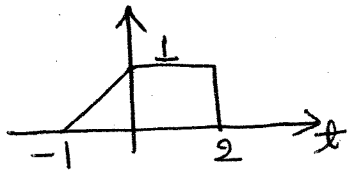


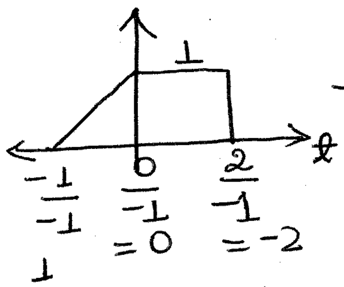



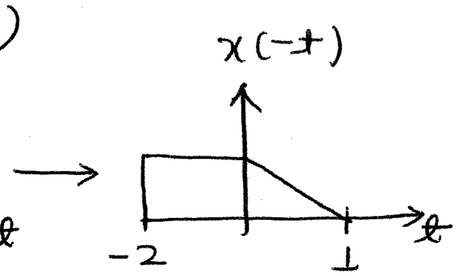

## 3). Time Reversal : Folding about y-axis

$a = -1$

$x(t) \longrightarrow x(-t) = x(at)$

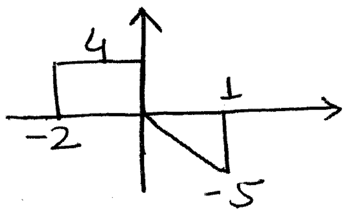


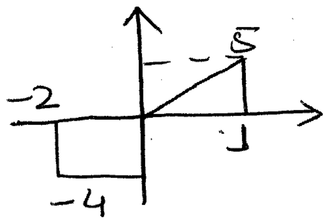



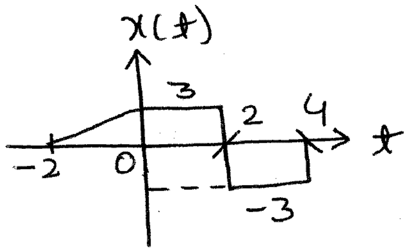

## 4). Amplitude-reversal : Folding about x-axis

$x(t) \longrightarrow -x(t)$



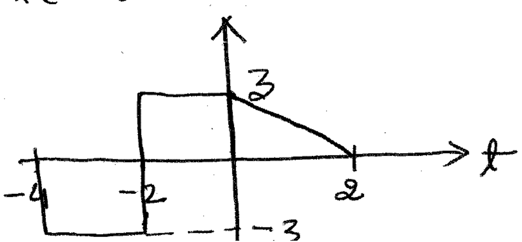




Q.N-:

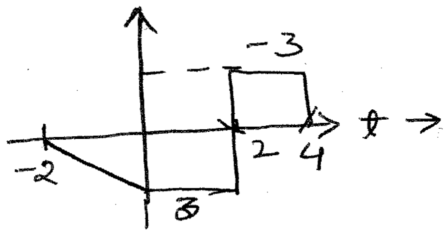



Draw (i) $x(-t)$
(ii) $-x(t)$

(i) $x(-t)$



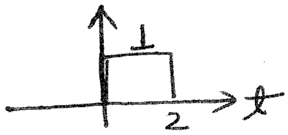

(ii) $-x(t)$



## 5). Amplitude-Shifting

Amplitude-Shifting → upward, downward

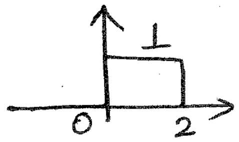
$$x(t) \longrightarrow y(t) = K + x(t)$$

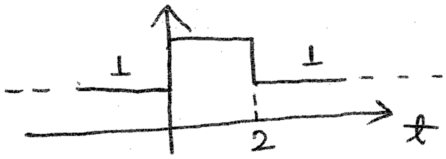



**case (i): when $K > 0$ : upward shifting**

Ex: $K = 1$

$$x(t) \longrightarrow y(t) = 1 + x(t)$$



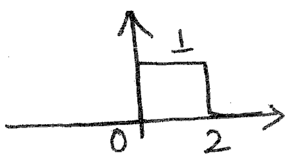

$$x(t) = \begin{cases} 0, & t < 0 \\ 1, & 0 < t < 2 \\ 0, & t > 2 \end{cases}$$

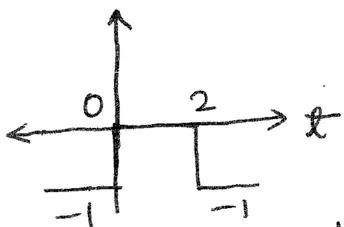
$$y(t) = \begin{cases} 1+0, & t < 0 \\ 1+1, & 0 < t < 2 \\ 1+0, & t > 2 \end{cases} = \begin{cases} 1, & t < 0 \\ 2, & 0 < t < 2 \\ 1, & t > 2 \end{cases}$$

**case (ii): when $K < 0$ : downward shifting**

Ex: $K = -1$

$$x(t) \longrightarrow y(t) = -1 + x(t)$$

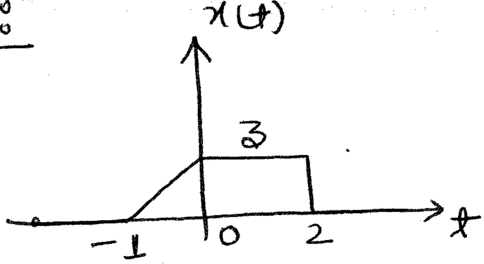



$$x(t) = \begin{cases} 0, & t < 0 \\ 1, & 0 < t < 2 \\ 0, & t > 2 \end{cases} \qquad y(t) = \begin{cases} -1+0, & t < 0 \\ -1+1, & 0 < t < 2 \\ -1+0, & t > 2 \end{cases}$$

$$= \begin{cases} -1, & t < 0 \\ 0, & 0 < t < 2 \\ -1, & t > 2 \end{cases}$$

Q.N :-

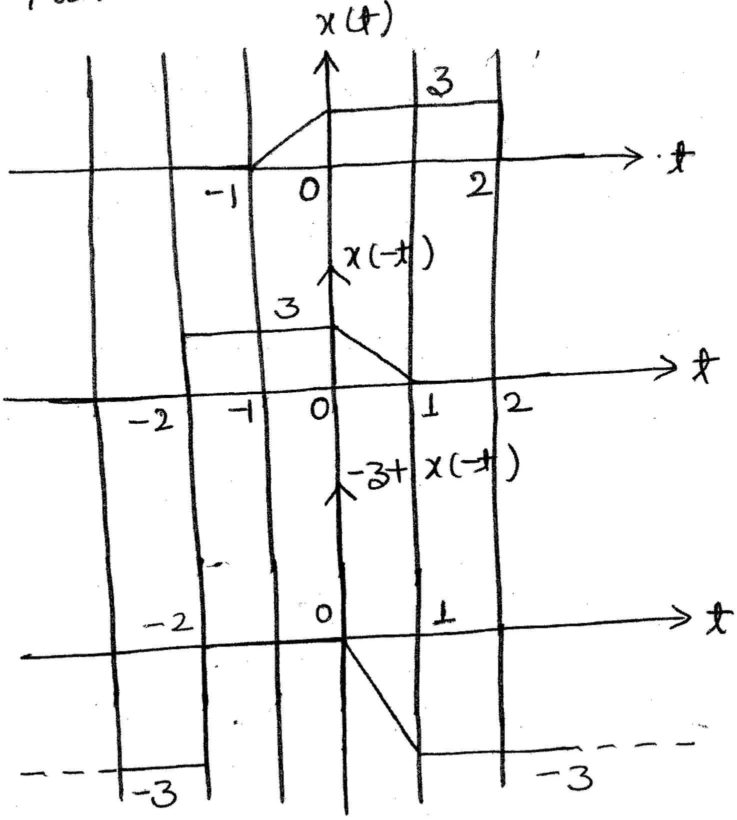



draw y(t) where

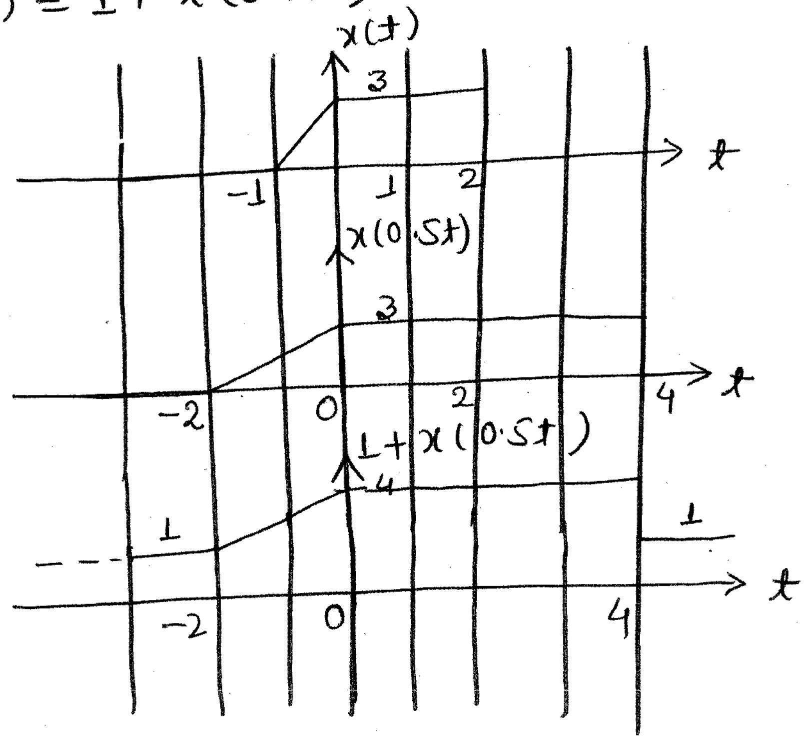
(i) $y(t) = -3 + x(-t)$

(ii) $y(t) = 1 + x(0.5t)$

Solution : (i) $y(t) = -3 + x(-t)$




(ii) $y(t) = 1 + x(0.5t)$

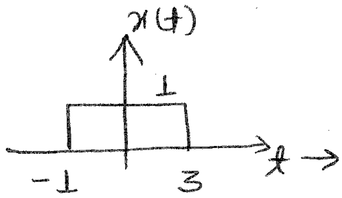



$\frac{\cancel{2}0}{0.\cancel{5}} = 4$ $\frac{\cancel{1}0}{0.\cancel{5}} = 2$

$\frac{\cancel{-10}}{\cancel{0.5}} = -2$

$\frac{0}{0.5} = 0$

Q.N -:




Draw signal $y(t)$ if

$$y(t) = x(2t+3)$$

**NOTE :-**

1. $x(2t) \xrightarrow[]{\text{left shift} = 1} x[2(t+1)]$    $t \to (t+1)$

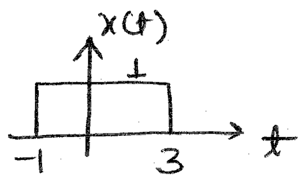
2. $x(-3t) \xrightarrow[t \to (t-2)]{\text{Right-shift} = 2} x[-3(t-2)] = x[-3t+6]$

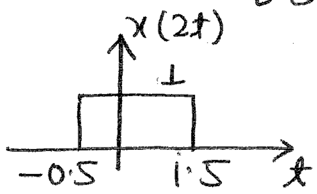
3. $x(t+1) \xrightarrow[\substack{\text{perform time} \\ \text{scaling by } 2 \\ t \to 2t}]{} x(2t+1)$

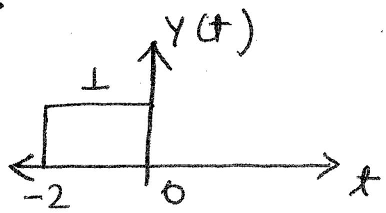
Ex -: $x(t-4) \xrightarrow[\substack{\text{perform time scaling} \\ \text{by } (-0.5) \\ t \to -0.5t}]{} x(-0.5t-4)$

Solution -: 1st method : $y(t) = x(2t+3) = x[2(t+1.5)]$

$$x(t) \xrightarrow{\text{time scaling}} x(2t) \xrightarrow[\text{L.S.} = 1.5]{\substack{t \to (t+1.5) \\ \text{time-shifting}}} y(t) = x[2(t+1.5)]$$

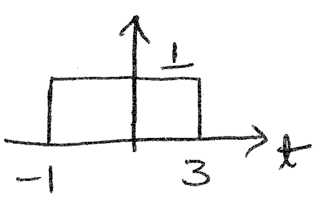


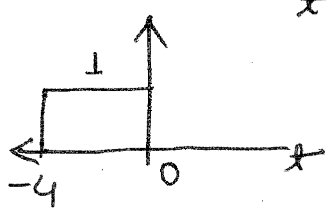



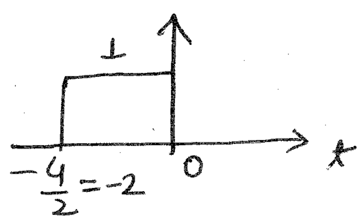




2nd method : $y(t) = x(2t+3)$

$$x(t) \xrightarrow{\text{time shifting}} x(t+3) \xrightarrow[t \to 2t]{\substack{\text{by } 2 \\ \text{time scalling}}} x(2t+3)$$



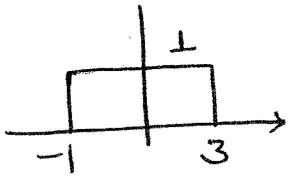




$$x(t) \longrightarrow x(t+1.5) \longrightarrow x(2t+1.5) \neq y(t)$$

## 3rd method (Trick):

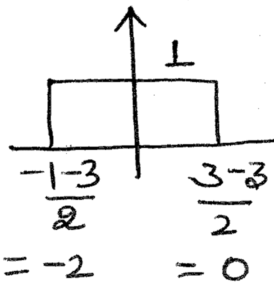
Imp

$x(t)$



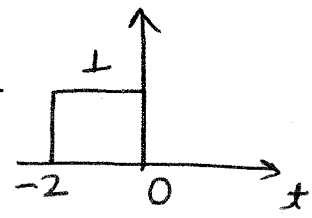

=

$x(2t+3)$ → draw against w.r.t. $t$.




$\frac{(\cancel{2}t+\cancel{3})-\cancel{3}}{\cancel{2}} = t$

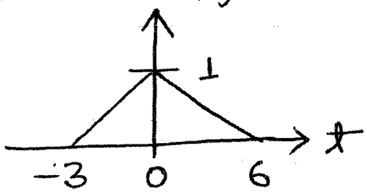
= $x(2t+3)$




NOTE: when in any question is given that relate to fourier transform & laplace transform then this trick is not applicable. So for this 1st & 2nd method is applicable only.

Q.N:

$x(t)$



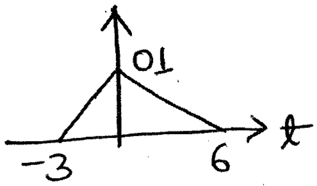

draw signal $y(t)$ if

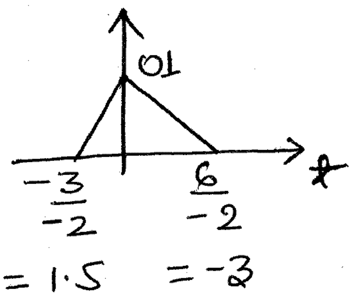
$y(t) = x(-2t+1)$

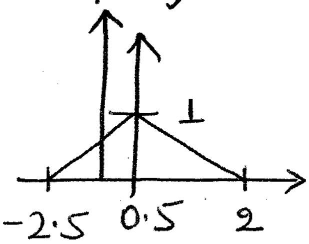
Soln:

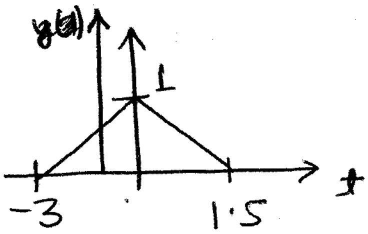
**1st method:** $y(t) = x(-2t+1) = x[-2(t-0.5)]$ → Right shifting $= 0.5$

$x(t) \xrightarrow{\text{time scaling}} x(-2t) \xrightarrow{\text{time shifting}} y(t)$



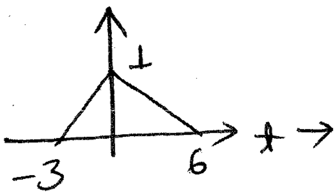



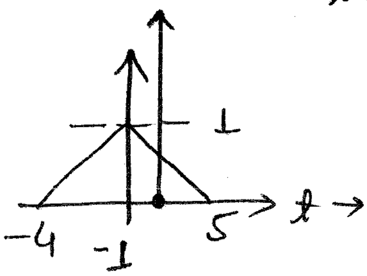



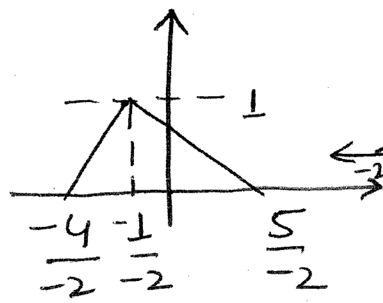



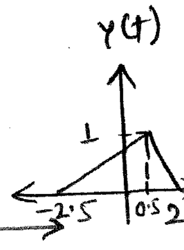

**2nd method:** $y(t) = x(-2t+1)$

$x(t) \xrightarrow[t \to (t+1)]{\text{time shifting}} x(t+1) \xrightarrow[x(-2t+1)]{\text{time scaling}} x(-2t+1) = y(t)$

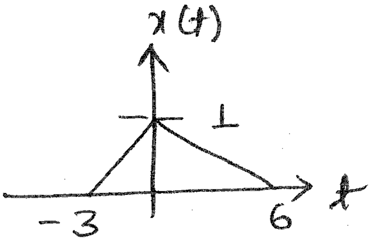


## 3rd method (Trick) :

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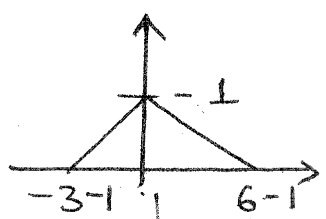
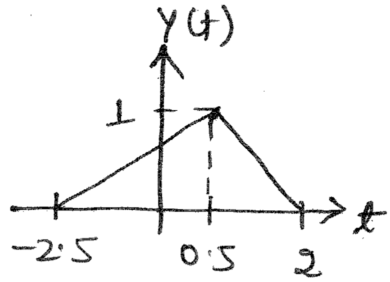
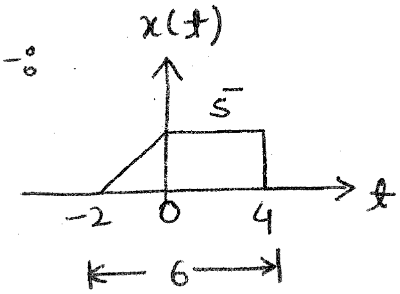


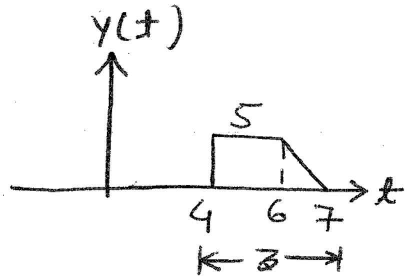

$$\frac{-2t+1-1}{-2} = t$$

$\frac{-3-1}{-2} = +2$ $\quad \frac{0-1}{-2} = 0.5 \quad \frac{6-1}{-2} = -2.5$




Q.N :-



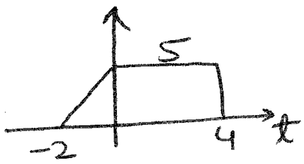



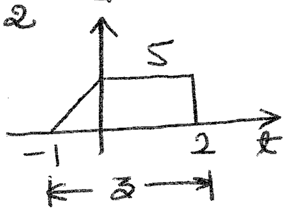

Find y(t) in term of x(t).

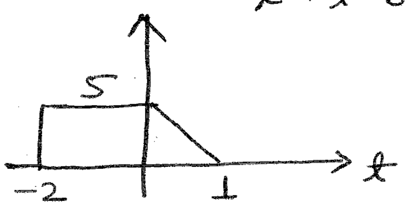
Sol$^n$ :- 1st method :

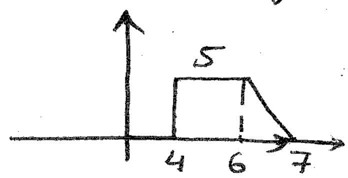
$$x(t) \longrightarrow x(2t) \xrightarrow{t \to -t} x(-2t) \xrightarrow[t \to t-6]{R.S.=6} x[-2(t-6)]$$
$$= x(-2t+12)$$

Duration : 6 $\longrightarrow \frac{6}{2} = 3$



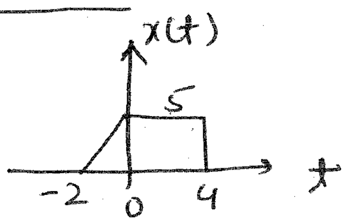



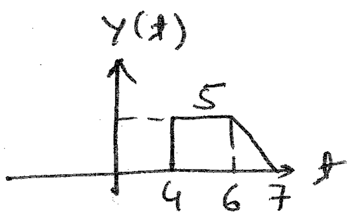




## 2nd method :

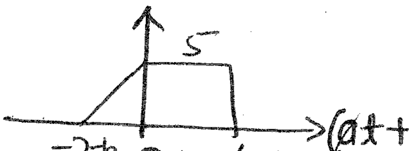
Imp




$y(t) = x(at+b)$ , $a = ?$, $b = ?$

$x(at+b) \rightarrow$ (t)




$\frac{-2-b}{a} \quad \frac{0-b}{a} \quad \frac{4-b}{a} \qquad \frac{(\cancel{at}+\cancel{b})-\cancel{b}}{\cancel{a}} = t$

$$\frac{-2-b}{a} = 7 \;\&\; \frac{4-b}{a} = 4$$
$$-2-b = 7a \;\&\; 4-b = 4a$$
$$-2 = 7a+b \;\&\; 4 = 4a+b$$
$$-6 = 3a \qquad \boxed{b = +12}$$
$$\boxed{a = -2} \qquad \boxed{y(t) = x(-2t+12)}$$