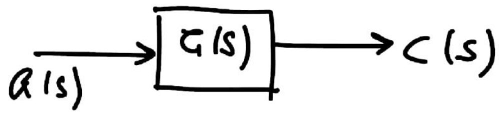
Introduction to Control Systems

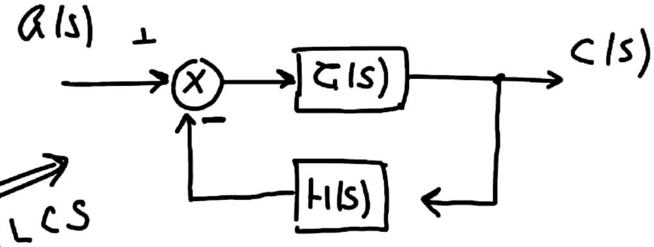
Comprehensive Course on Control Systems - Part I

Aditya Kanwal • Lesson 1 • July 7, 2021

CONTROL SYSTEMS :-



OLCS



CLCS

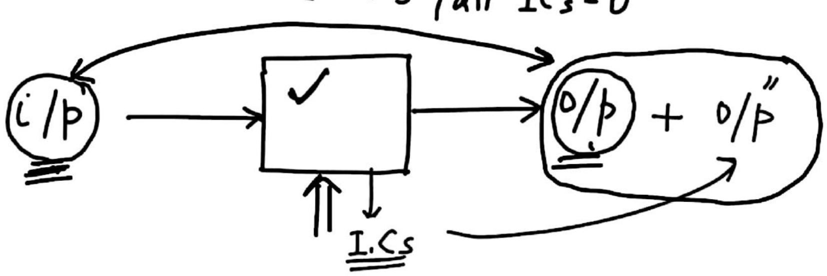
Mathematical model. ⇒ M.M

→ Transfer function:- It is M.M of a physical system.

① It is the ratio of the LT of the o/p to the LT of the i/p with all initial conditions = 0.

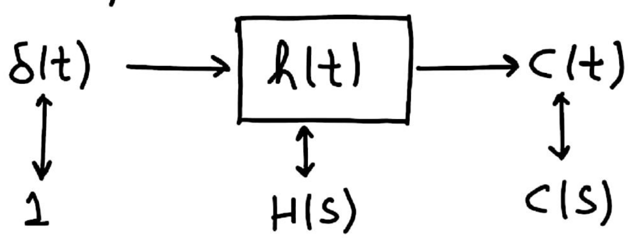
~~Linear~~ TF is defined only for LTI.

$$T(s) = \left.\frac{LT[o/p]}{LT[i/p]}\right|_{\text{all } ICs = 0} = \frac{C(s)}{R(s)}$$



$$TF = \frac{LT[c(t)]}{LT[r(t)]} \neq LT\left[\frac{c(t)}{r(t)}\right]$$

2) TF can also be defined as the LT of the impulse response of the system. with all IC = 0

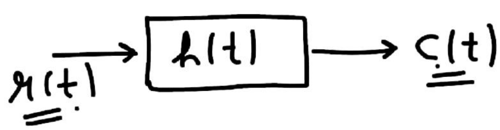


$$\underline{H(s)} = \frac{C(s)}{1} = \underline{C(s)}$$

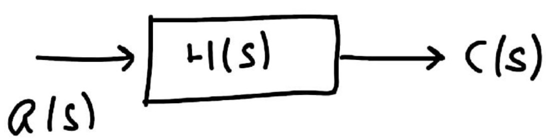
$$c(t) = \delta(t) \otimes h(t)$$

$$c(t) = h(t)$$

$$\boxed{C(s) = H(s)}$$

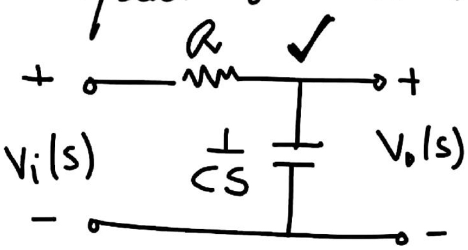


$$\Rightarrow \quad c(t) = r(t) \otimes h(t)$$



$$\Rightarrow \quad C(s) = R(s) \cdot H(s)$$

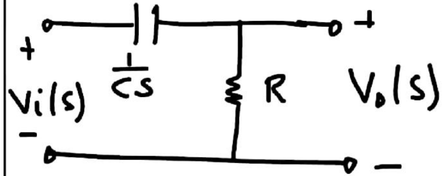
→ The order of the T.F of an electrical n/w is equal to the total no. of energy storage elements.



$$V_o(s) = \frac{V_i(s)}{R + \frac{1}{CS}} \cdot \frac{1}{CS}$$

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{1}{1 + RCS}}$$

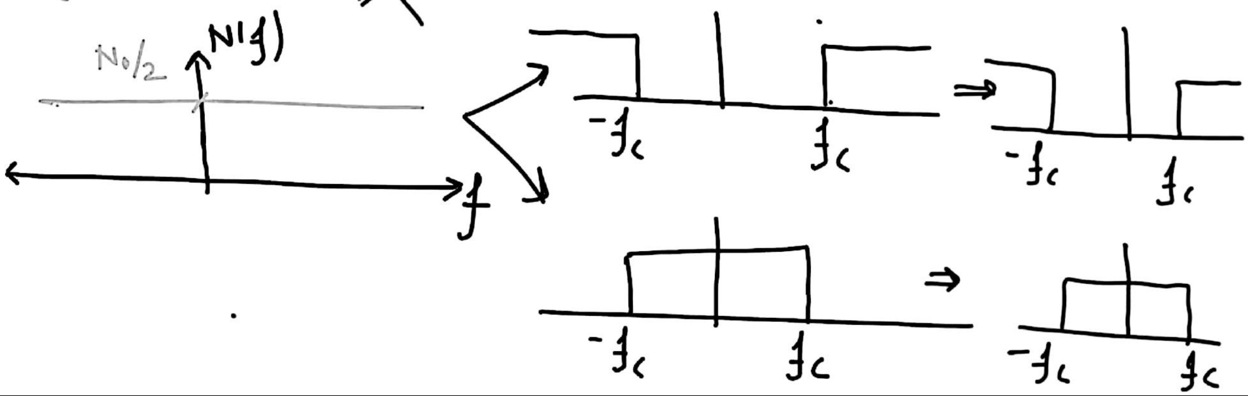
order − 1



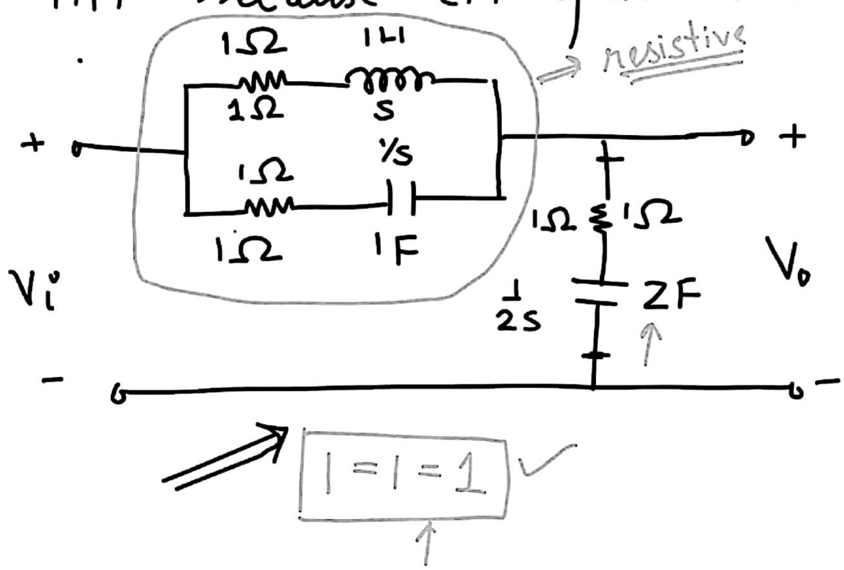
$$V_o(s) = \frac{V_i(s)}{R + \frac{1}{Cs}} \cdot R$$

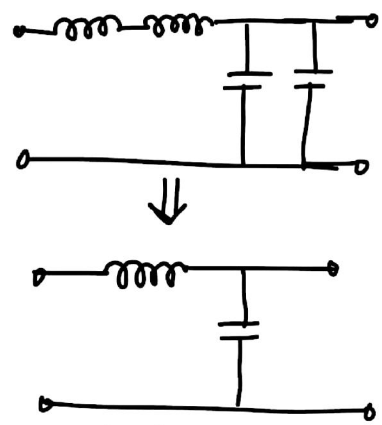
$$\frac{V_o(s)}{V_i(s)} = \frac{RCs}{1 + RCs}$$

✓ LPF and ~~HPF~~



∘ In Control system a LPF is preferred over an HPF because LPF passes less noise in the system.



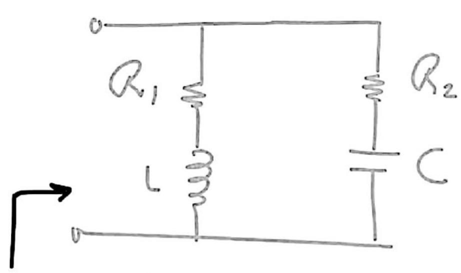


$$V_o(s) = \frac{V_i(s)}{\left[(1+s)\,||\,\left(1+\frac{1}{s}\right)\right] + \left(1+\frac{1}{2s}\right)}\left(1+\frac{1}{2s}\right)$$

$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{2s+1}{2s}}{\frac{(1+s)\left(1+\frac{1}{s}\right)}{1+s+1+\frac{1}{s}} + 1+\frac{1}{2s}} = \frac{\frac{2s+1}{2s}}{1 + 1+\frac{1}{2s}}$$

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{2s+1}{4s^{1}+1}}$$

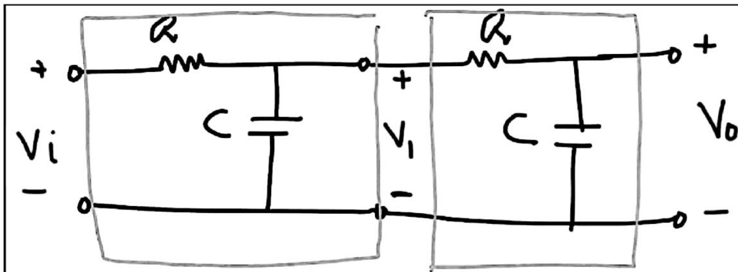
Order — 1



$Z \Rightarrow$ purely resistive

This circuit will be at resonance at any freq. provided

$$\boxed{R_1^2 = R_2^2 = \frac{L}{C}}$$ ✓



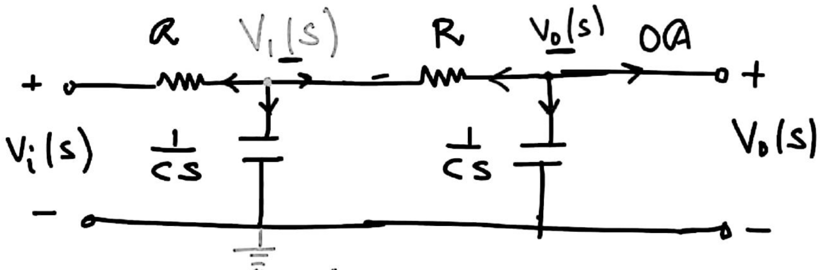
$$A_{v_1} = \frac{V_1}{V_i} \qquad A_{v_2} = \frac{V_o}{V_1}$$

$$A_v = A_{v_1} \cdot A_{v_2}$$

$$= \frac{1}{1+RCs} \cdot \frac{1}{1+RCs}$$

$$= \frac{1}{(1+RCs)^2}$$

~~$$= \frac{1}{R^2C^2s^2 + 2RCs + 1}$$~~



$$\frac{V_o(s)}{V_i(s)} = ?$$

$$\frac{V_1(s) - V_i(s)}{R} + \frac{V_1(s)}{1/Cs} + \frac{V_1(s) - V_o(s)}{R} = 0$$

$$V_1(s)\left[\frac{1}{R} + Cs + \frac{1}{R}\right] = \frac{V_i(s)}{R} + \frac{V_o(s)}{R}$$

$$V_1(s)\left[\frac{2 + RCs}{\cancel{R}}\right] = \frac{V_i(s) + V_o(s)}{\cancel{R}}$$

$$V_1(s) = \frac{V_i(s) + V_o(s)}{2 + RCS} \quad \text{——} \ (1)$$

$$\frac{V_o(s) - V_1(s)}{R} + \frac{V_o(s)}{1/CS} = 0$$

$$V_o(s)\left[\frac{1}{R} + CS\right] = \frac{V_1(s)}{R}$$

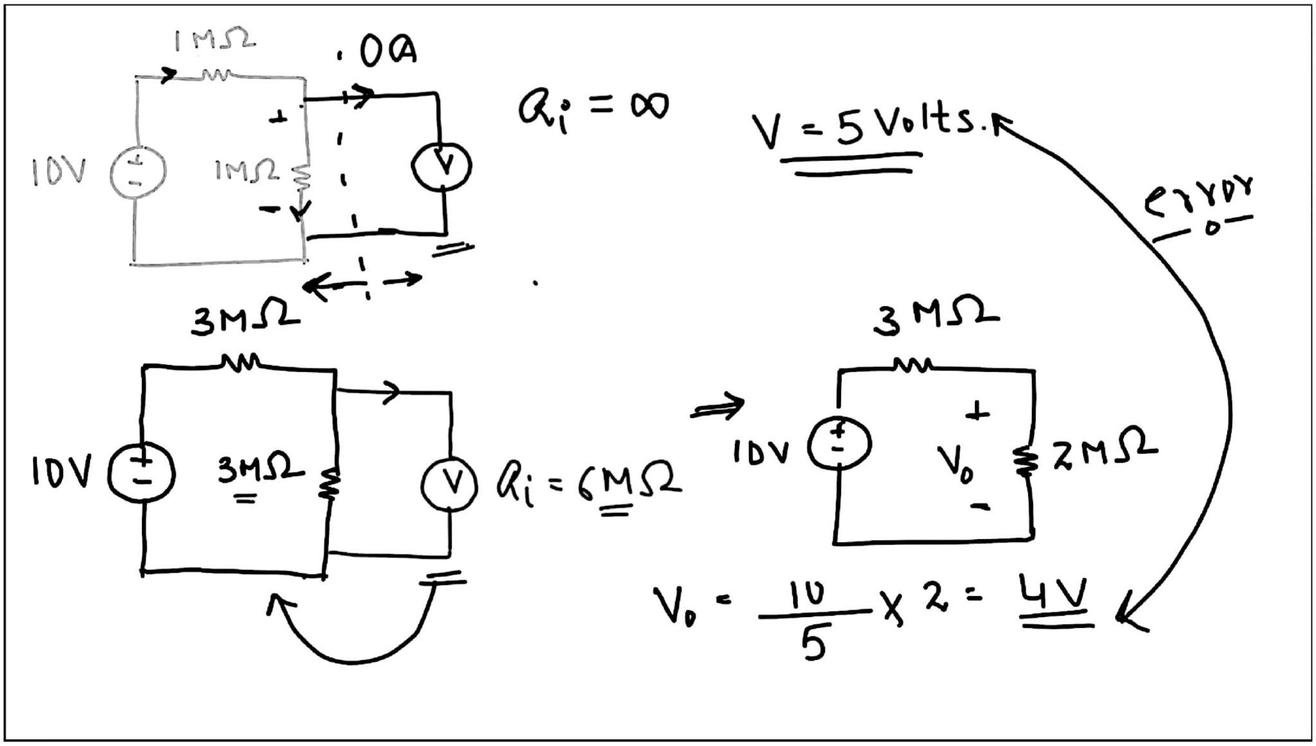
$$V_o(s)\left[\frac{1 + RCS}{\cancel{R}}\right] = \frac{1}{\cancel{R}}\left[\frac{V_i(s) + V_o(s)}{2 + RCS}\right]$$

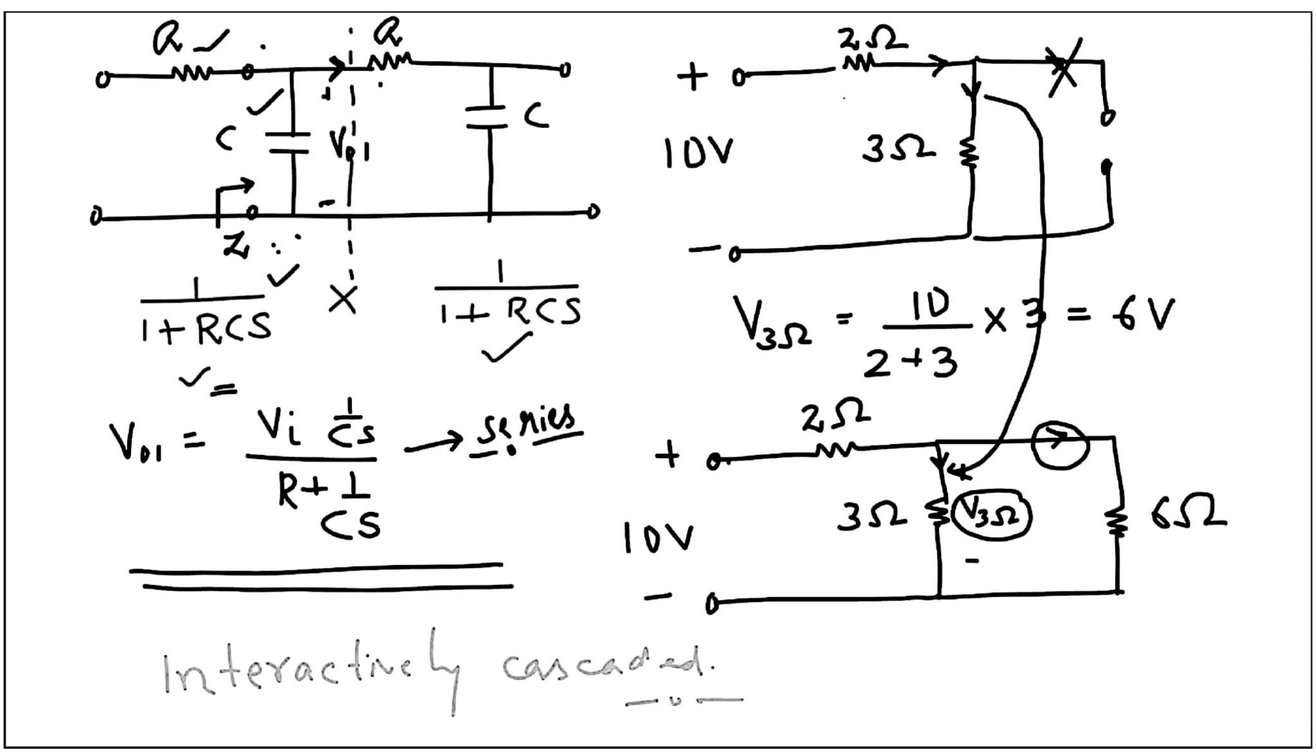
$$V_o(s)\left[(1 + RCS)(2 + RCS)\right] = V_i(s) + V_o(s)$$

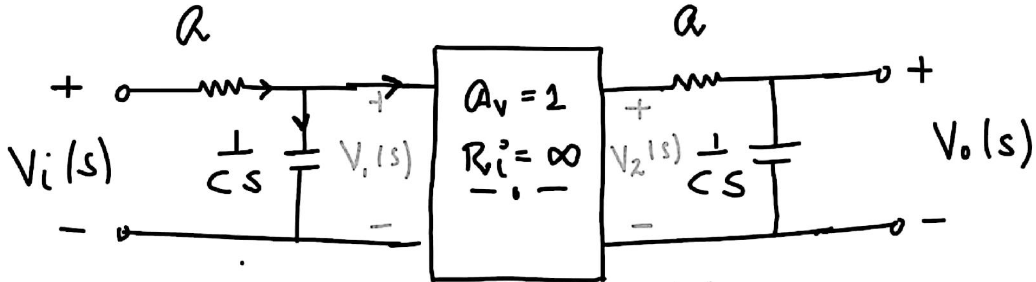
$$V_o(s)\left[2 + RCS + 2RCS + R^2C^2S^2 - 1\right] = V_i(s)$$

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{1}{R^2C^2S^2 + 3RCS + 1}}$$

@adityakanwalEx Ies


1MΩ
.0A
10V
1MΩ
Ri = ∞
V = 5 Volts.
error
3MΩ
10V
3MΩ
Ri = 6MΩ
3MΩ
10V
V0
2MΩ
V0 = 10/5 x 2 = 4V


R
R
C
C
V01
Zi
1/(1+RCS)
1/(1+RCS)
V01 = Vi (1/CS) / (R + 1/CS) → series
Interactively cascaded.
2Ω
10V
3Ω
V3Ω = 10/(2+3) x 3 = 6V
2Ω
10V
3Ω
V3Ω
6Ω



$$\frac{V_o(s)}{V_i(s)} = \frac{V_o(s)}{V_2(s)} \cdot \frac{V_2(s)}{V_1(s)} \cdot \frac{V_1(s)}{V_i(s)} .$$

$$= \frac{\frac{1}{Cs}}{R + \frac{1}{Cs}} \cdot 1 \cdot \frac{\frac{1}{Cs}}{R + \frac{1}{Cs}} = \frac{1}{1+RCs} \cdot \frac{1}{1+RCs}$$

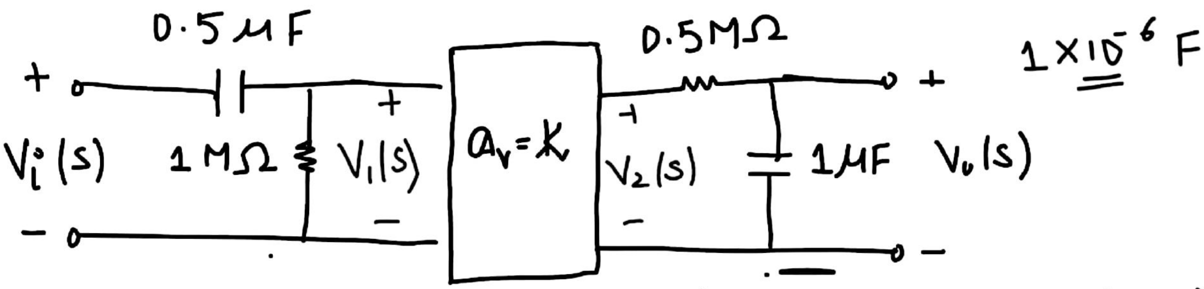
$$= \frac{1}{(1+RCs)^2} = \frac{1}{R^2C^2s^2 + 2RCs + 1}$$

When → non-interactively cascaded.

$$A_v = A_{v_1} \cdot A_{v_2} \cdot A_{v_3} \cdots$$

→ $\frac{1}{s+1}$ ⇒ 2 systems ⇒ non interactively cascade.

⇒ Final system T. F = ? $\frac{1}{s+1} \cdot \frac{1}{s+1} = \frac{1}{(s+1)^2}$



$$\frac{V_o(s)}{V_i(s)} = \frac{V_o(s)}{V_2(s)} \cdot \frac{V_2(s)}{V_1(s)} \cdot \frac{V_1(s)}{V_i(s)} \qquad \frac{V_o(s)}{V_i(s)} = \frac{K(0.5s)}{(1+0.5s)^2}$$

$$= \frac{\frac{1}{s\mu}}{0.5M + \frac{1}{s\mu}} \cdot K \cdot \frac{1M}{1M + \frac{1}{0.5\mu s}} = \frac{1}{1+0.5s} \cdot K \cdot \frac{0.5s}{1+0.5s}$$

Ex:- Find the T.F of a system whose impulse response is te^{-t}.

Sol: T.F = LT[IR] $\Rightarrow$ $T(s) = C(s) = \frac{1}{(s+1)^2}$

Ex:- Find the T.F of a system whose step response is $t.e^{-t}$.

Sol: $T(s) = \frac{C(s)}{R(s)}$ $\qquad C(s) = \frac{1}{(s+1)^2}$ $\qquad r(t) = u(t)$, $R(s) = \frac{1}{s}$

$$T(s) = \frac{s}{(s+1)^2}$$