

# Signals and Systems

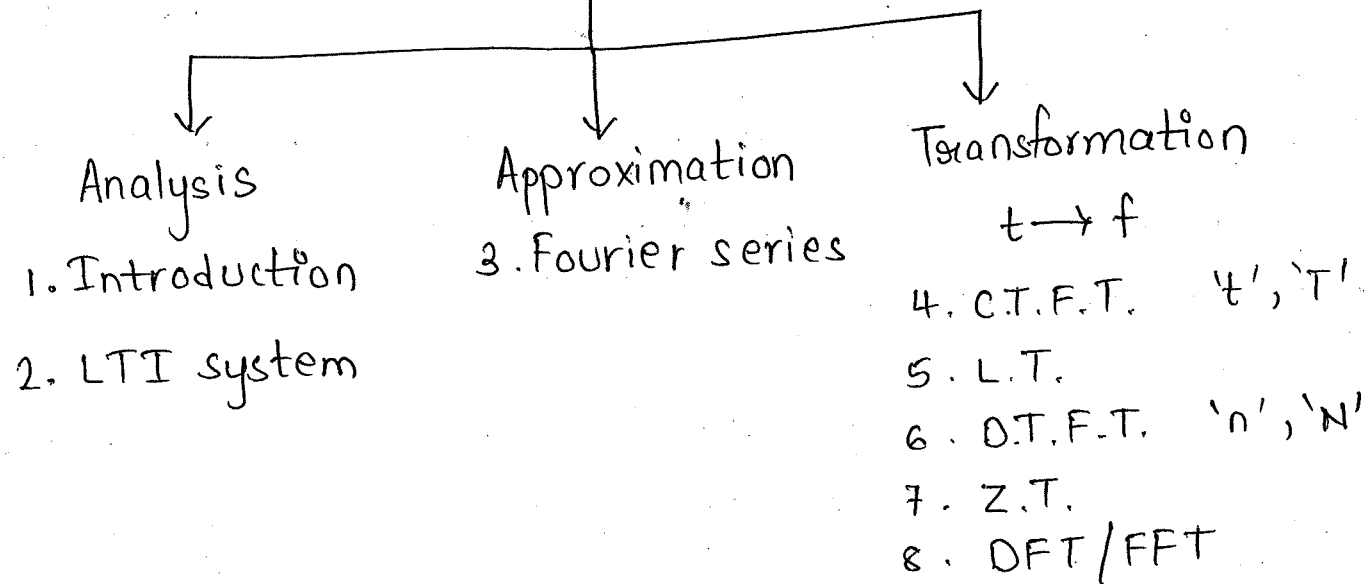
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GATE: (9 to 12 Marks)

Reference Book: S & S by Ranjan and HSU

Mentor: Sujal sir

## Signals and Systems



## 1.] Introduction to Signals and Systems

→ What is signal?

→ What is system?

→ Characteristics of signals.

→ Types of signals

→ Some standard signals

→ Transformation on signals  $x(t)$

① Time shifting  $x(t \pm t_0)$

② Time scaling  $x(\alpha t)$

③ Time reversal  $x(-t)$

④ Amplitude scaling  $k \cdot x(t)$

→ Problems

→ Classification of signals

→ Classification of systems

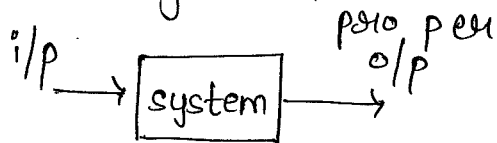
→ Summary

→ Problems

What is signal?

-It is the indication from which some amount of information is to be conveyed from one place to another.

What is system?



-System is nothing but group of elements/physical components arranged in such a way that it gives proper output to given input.

Eg:- A fan without blades: No air flow: not proper o/p: Not a system

A fan with blades: Air flow: proper o/p: It is system

If we add controller from controlling purpose then it is called CONTROL SYSTEM.

### \* Characteristics of signals

(1) Dimension

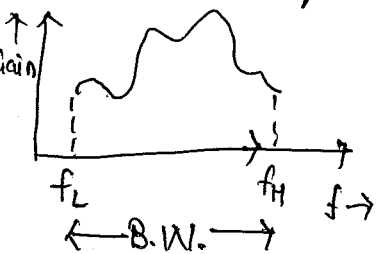
1D  $\rightarrow$  point (.) x-coordinate only

2D  $\rightarrow$  image  (x, y)

3D  $\rightarrow$  TV (x, y, t)

(2) Bandwidth

Range of frequencies occupy by signals.



(3) Randomness

more the randomness  
more the information

$$I = \log_2 \frac{1}{P_i} = -\log_2 P_i$$

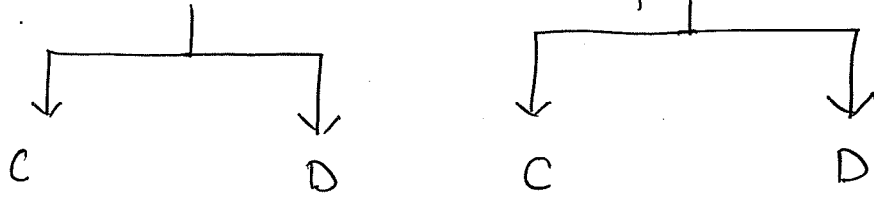
$$P_i = \frac{1}{8} \Rightarrow I = 3 \text{ bits}$$

$$\text{more random} \rightarrow P_i = \frac{1}{32} \Rightarrow I = 5 \text{ bits}$$

# \* Types of Signals \*

Based on

Time & Amplitude



(continuous) (discrete)

$t \rightarrow$  continuous  $\rightarrow t = 0.1, t = 0.01, t = 0.02, t = 2$

$n \rightarrow$  discrete  $\rightarrow n = 0, 1, 2, 3, 4, -1, -2, -3, -4$   
 $n = 1.5 \times$

$A \rightarrow$  continuous  $\rightarrow -\infty$  to  $+\infty$

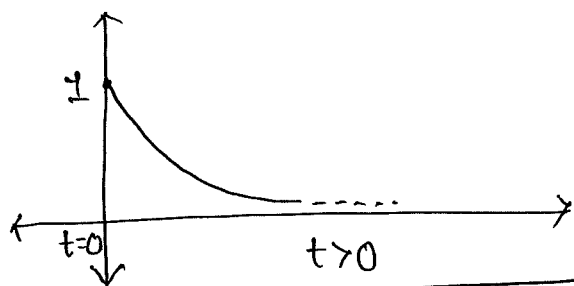
$A \rightarrow$  discrete  $\rightarrow [-2, -1, 0, 0.5, 2]$

## ① Continuous signal

time  $\rightarrow$  continuous, amplitude  $\rightarrow$  continuous

For eg:-  $x(t) = e^{-3t} \cdot u(t)$   
 $= e^{-3t} \cdot 1, t \geq 0$   
 $= 0, t < 0$

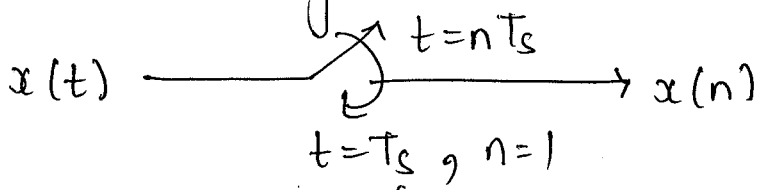
$u(t) = 1, t > 0$
$= 0, t < 0$
$= \text{not defined}, t = 0$



$$x(t) \Big|_{t \rightarrow \infty} = e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$$

A signal which is continuous in both amplitude and time at any instant amplitude and time is known as continuous signal.

② Discrete signal



$$x(t) = e^{-3t} \cdot u(t)$$

$$\downarrow t = nT_s$$

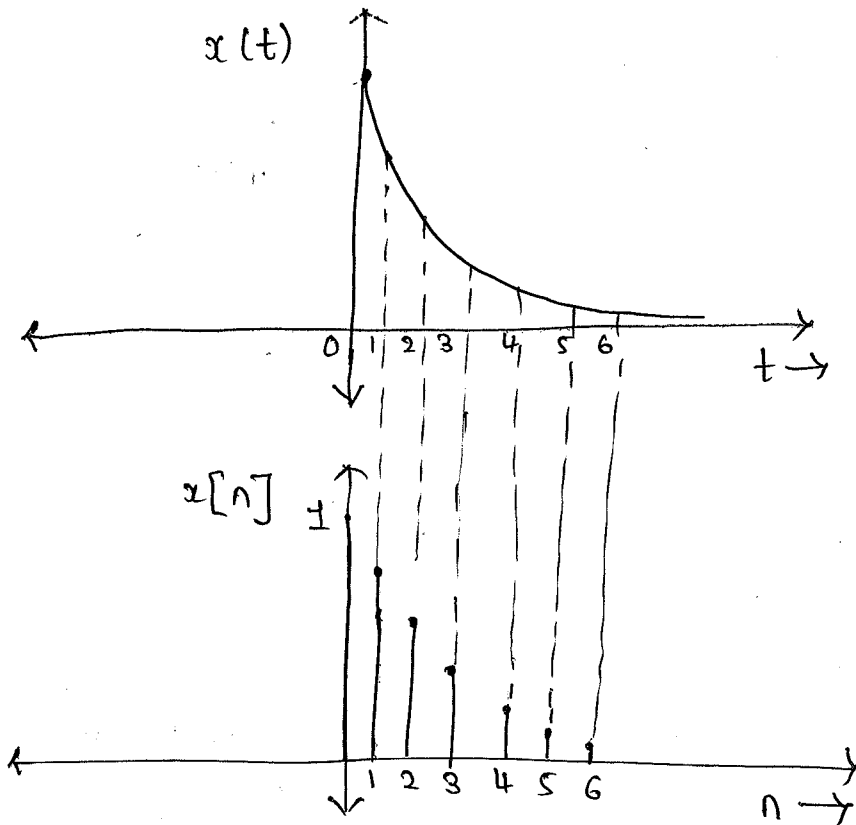
$$x(nT_s) = e^{-3nT_s} \cdot u[nT_s]$$

Let  $T_s = 1$

$$x[n] = e^{-3n} u[n], \quad n = 0, \pm 1, \pm 2, \pm 3, \dots$$

$$x[n] = e^{-3n}, \quad n \geq 0, \quad n = 0, \pm 1, \pm 2, \pm 3, \dots$$

$$= 0, \quad n < 0$$



If  $t = 10 \text{ sec}$   
 $t = 0, 2, 4, 6, 8$

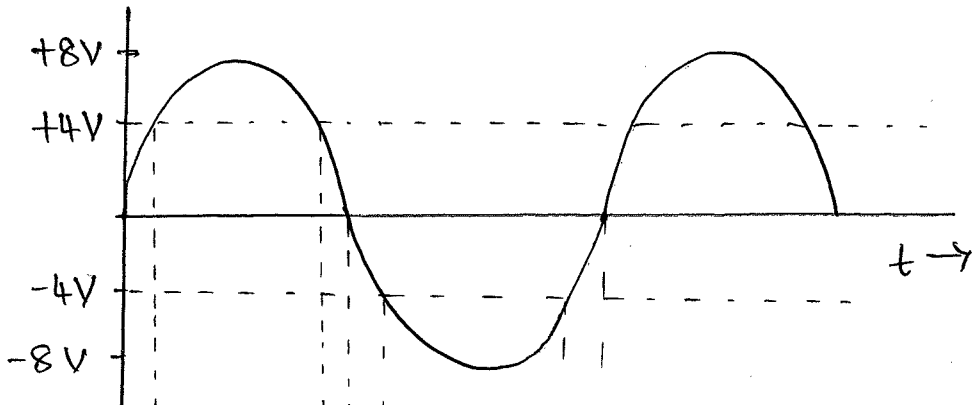
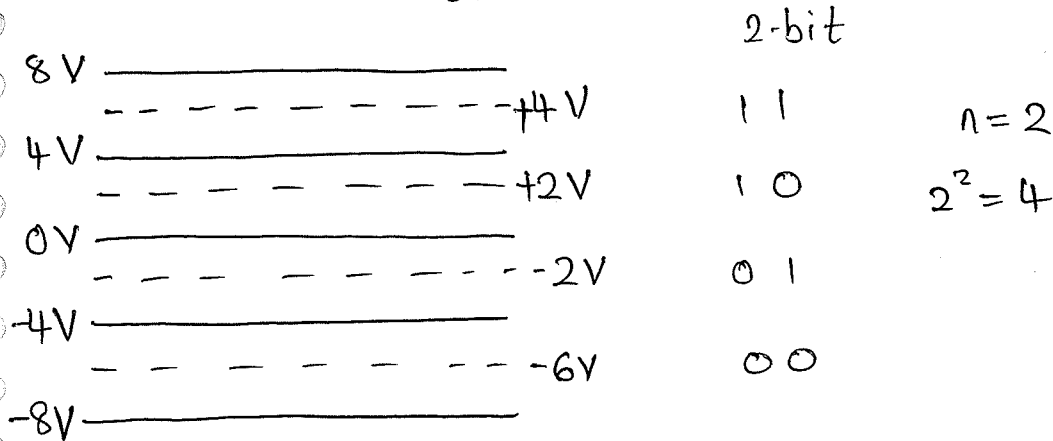
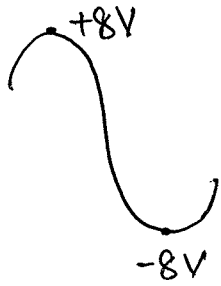
$$n = 5$$

$$T_s = 2$$

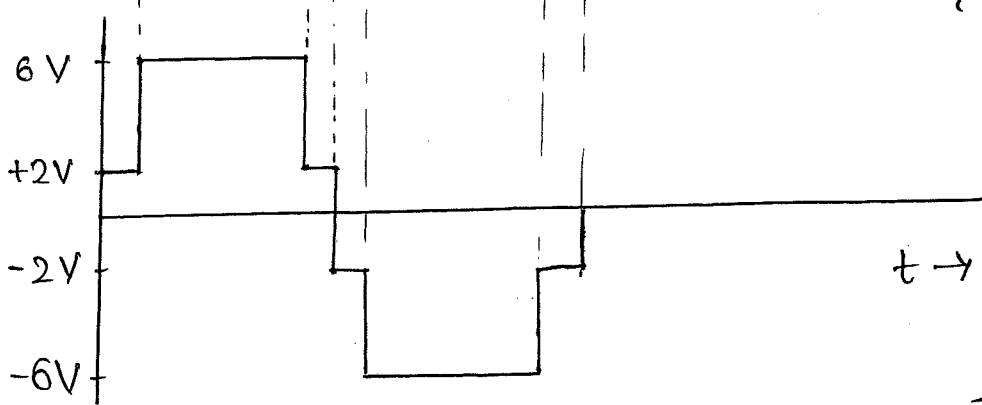
$$t = nT_s$$

### ③ Quantized Signal

- A signal which is continuous in time and discrete in amplitude is called quantized signal.



Range: [-6, -2, 2, 6]

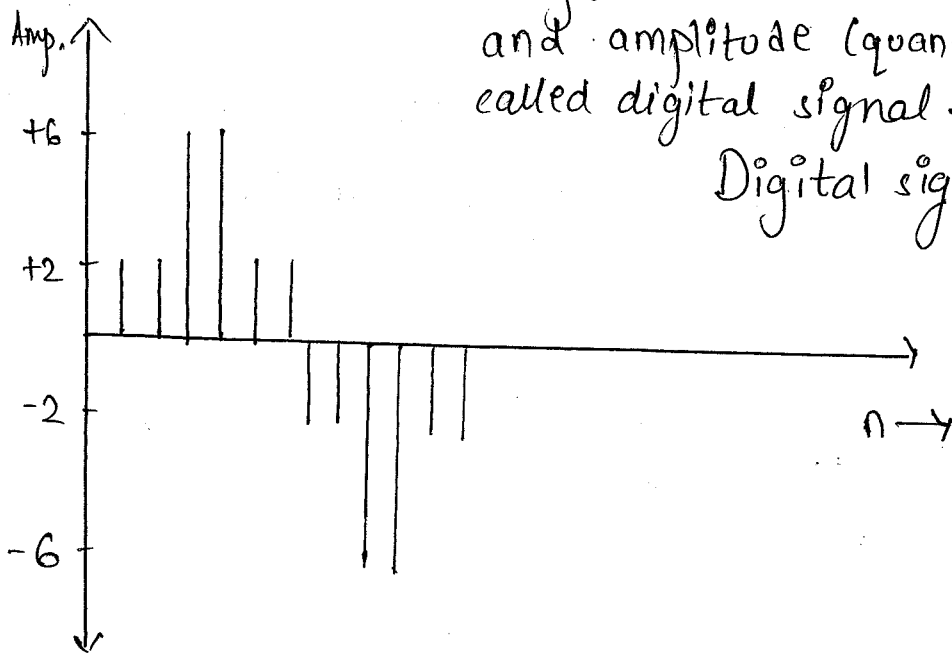


$t \rightarrow$  continuous

$A \rightarrow$  discrete

A signal which is discrete in time and amplitude (quantized amp.) is called digital signal.

Digital signal.



Types of signals	Amplitude	Time
Continuous time signal	C	C ↓ Sampling
Discrete time signal	C ↓ Quantization	D
Quantised signal	D	C
Digital signal	D	D

Amplitude

NOTE: ~~Time~~ conversion from C → D ⇒ Quantizer

Amplitude conversion from C → D ⇒ Sampling  
Time →

\* Some standard signals \*

[1] Continuous step signal

$$u(t) = A, t > 0$$

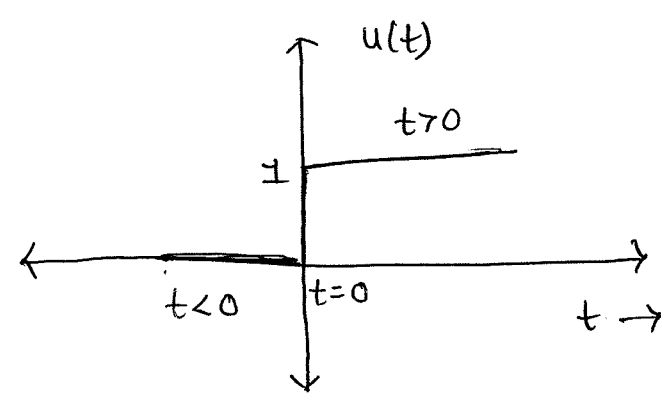
$$= 0, t < 0$$

if  $A=1 \Rightarrow$  unit step signal

$$u(t) = 1, t > 0$$

$$= 0, t < 0$$

$$= \text{not defined}, t = 0$$



Practically  $u(t)|_{t=0} = 1/2$

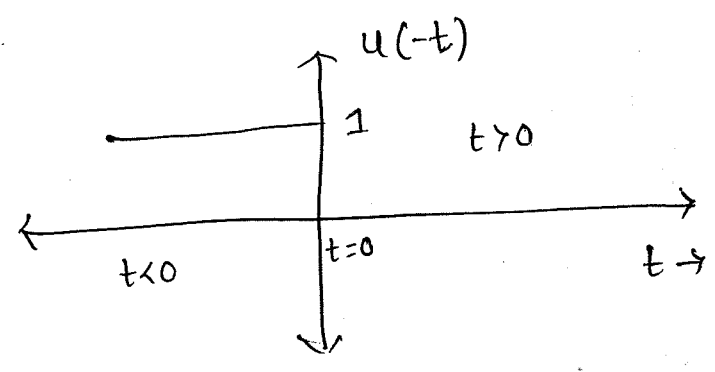
①  $u(-t) = 1$

$$u(-t) = 0, -t < 0$$

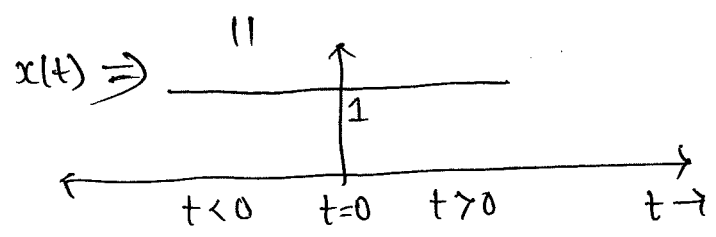
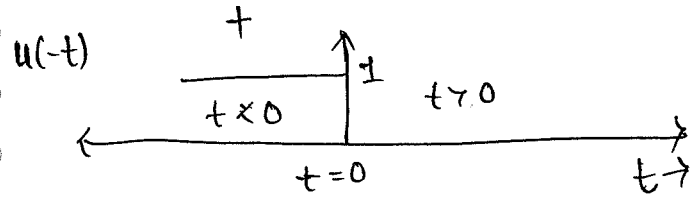
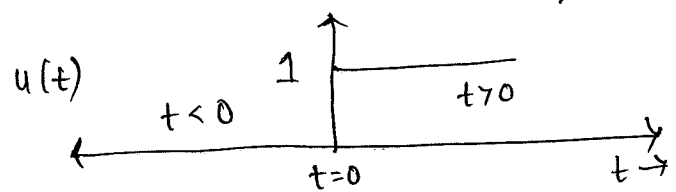
$$= 1, -t > 0$$

$$u(-t) = 1, t < 0$$

$$= 0, t > 0$$



②  $u(t) + u(-t) = x(t) = p = 1, \forall t$



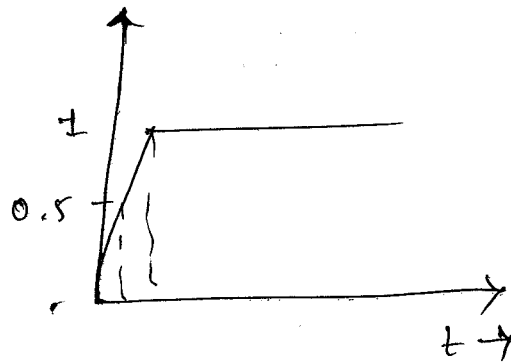
$$u(t) + u(-t) = 1$$

$$u(0) + u(0) = 1$$

$$2u(0) = 1$$

$$u(0) = \frac{1}{2}$$

$$u(t) \Big|_{t=0} = \frac{1}{2}$$



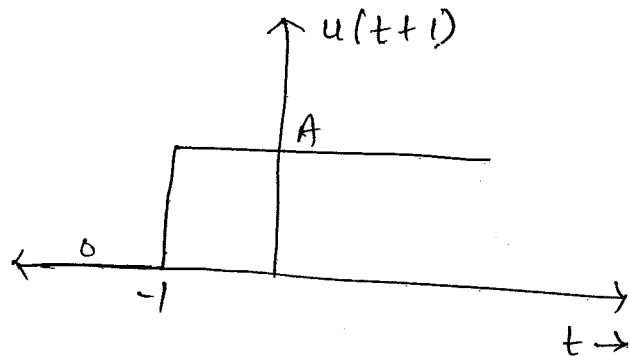
$$(4) u(t+1)$$

$$u(t+1) = 1, t+1 > 0$$

$$= 0, t+1 < 0$$

$$u(t+1) = 1, t > -1$$

$$= 0, t < -1$$



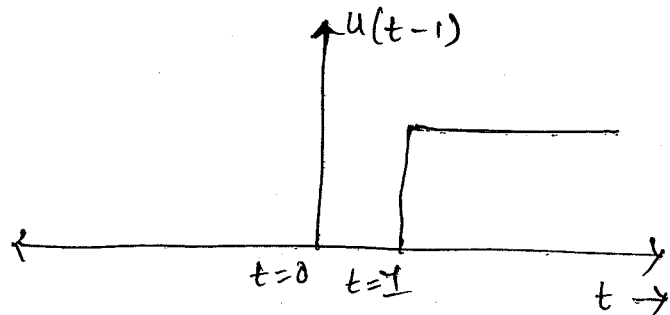
$$(5) u(t-1)$$

$$u(t-1) = 1, t-1 > 0$$

$$= 0, t-1 < 0$$

$$u(t-1) = 1, t > 1$$

$$= 0, t < 1$$



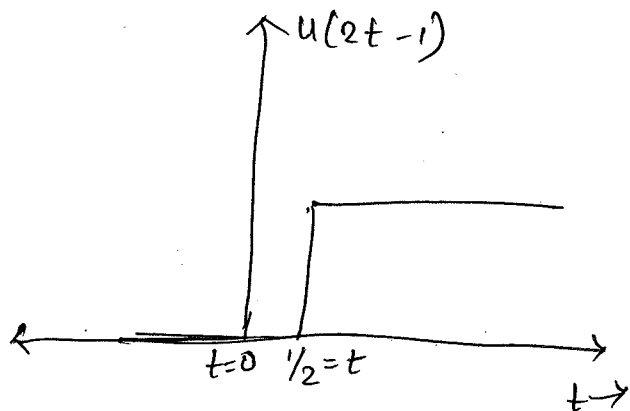
$$(6) u(2t-1)$$

$$u(2t-1) = 1, 2t-1 > 0$$

$$= 0, 2t-1 < 0$$

$$u(2t-1) = 1, t > \frac{1}{2}$$

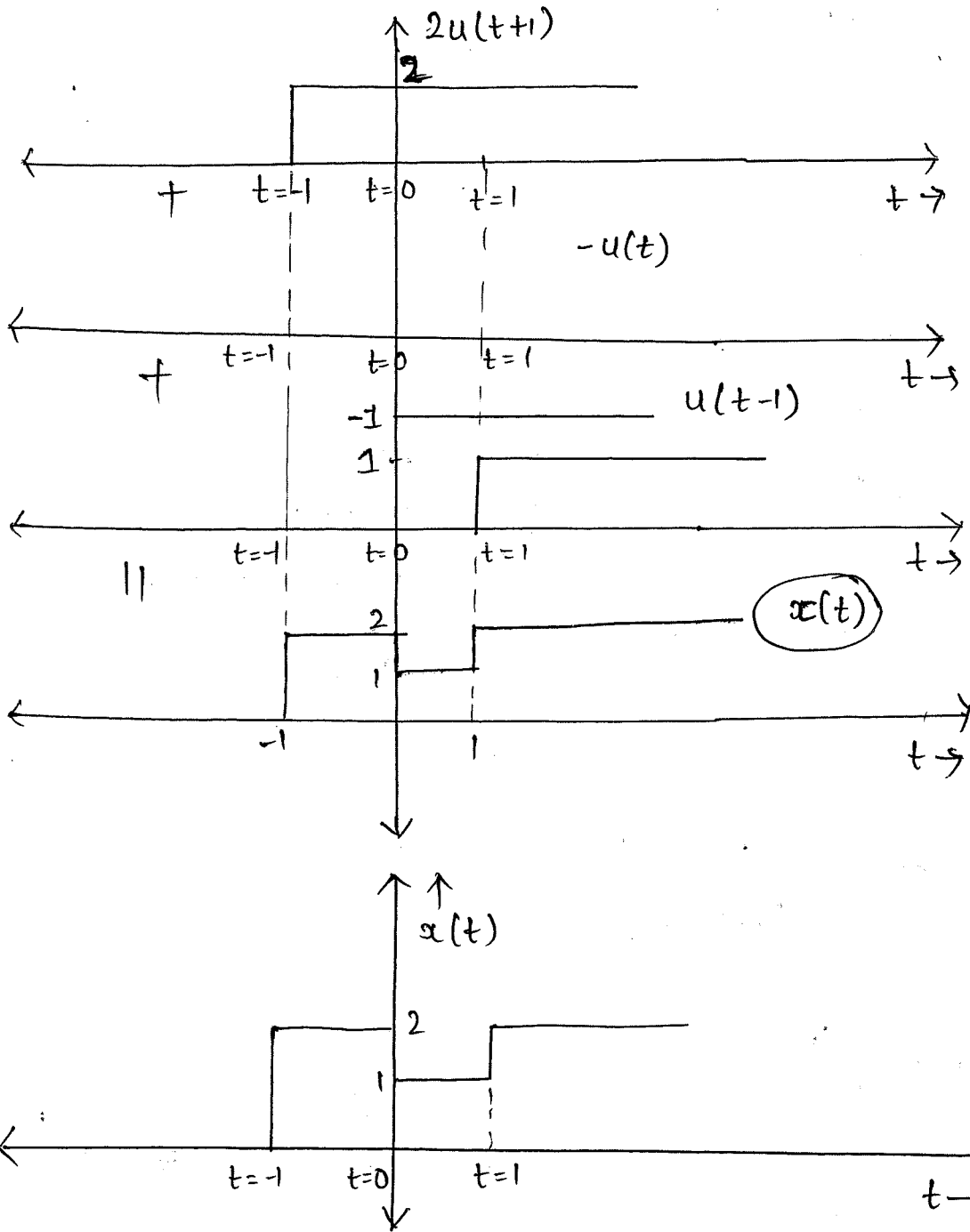
$$= 0, t < \frac{1}{2}$$





● Examples

● (1)  $x(t) = 2u(t+1) - u(t) + u(t-1)$

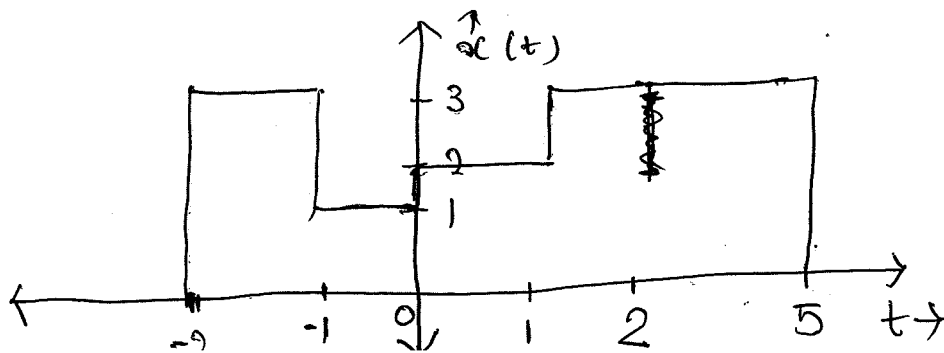


(2)  $x(t) = u(t-2) + u(t) + 3u(t+2) - 2u(t+1) - 3u(t-5)$

$t = 2$        $t = 0$        $t = -2$        $t = 1$        $t = 5$

Arrange in ascending order

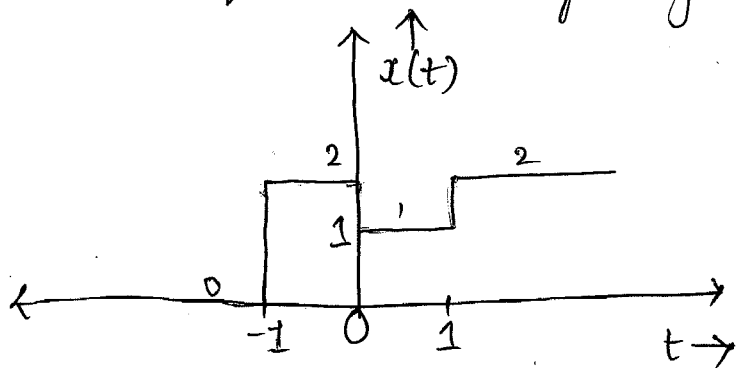
$= 3u(t+2) - 2u(t+1) + u(t) + u(t-2) - 3u(t-5)$



From graph to equation:-

NOTE: Change in amplitude = Next amp. - Previous amp.

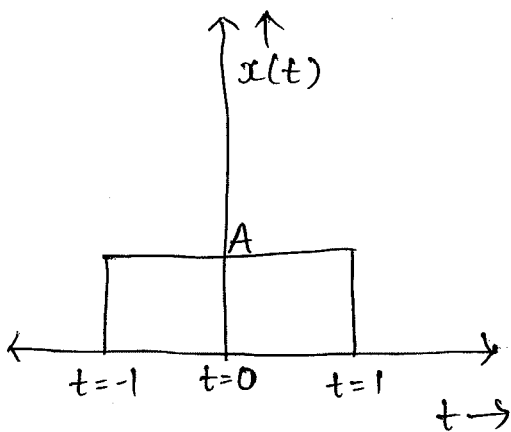
Q:- Write eq<sup>n</sup> of following signal?



- ① First mark sudden change points
- ② Amplitude check
- ③ Write individual signal eq<sup>n</sup>.

Sol<sup>n</sup>:  $2u(t+1) - u(t) + 1 \cdot u(t-1)$

$x(t)$

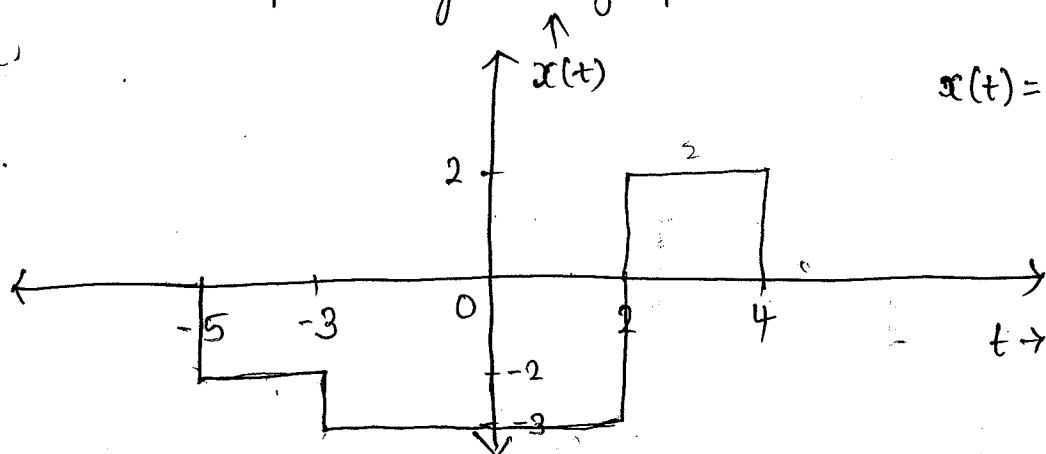


$$x(t) = A \cdot u(t+1) - A u(t-1)$$

$$x(t) = A (u(t+1) - u(t-1))$$

NOTE: Whenever there is sudden change in signal  $\rightarrow$  step signal exists.

Q:- Write eq<sup>n</sup> for given graph:- (in terms of step signal)



$$x(t) = -2u(t+5) - 1u(t+3) + 5u(t-2) - 2u(t-4)$$