unacademy

Comprehensive Course on Digital Electronic Circuit for 2021

Laws and Theorems in Boolean Algebra

Lesson 1 • June 11, 2020 • Sonu Lal Gupta

#. Boolean Algebra ⇒ 2-Marks

Logic Gates ⇒ BJT | MOSFET | Diode | Switching ⇒ 1 marks

Combinational ckt ⇒ 2 marks

Coding s/m | Number | 2's complement ⇒ ~~2~~ marks

Sequential ⇒ 2 Marks

ADC - DAC ⇒ 1 marks   6-8 Marks   2018 ⇒ 13 Marks Digital

#. Laws & theorem

① Idempodent Law :- $\boxed{A\cdot\bar{A}=0}$

② Commulative Law :- $AB = BA$, $A+B = B+A$, $A\oplus B = B\oplus A$

③ Associative Law :-

$A+(B+C) = (A+B)+C = A+B+C$

④ Distributive Law :-

$A(B+C) = AB+AC$

---

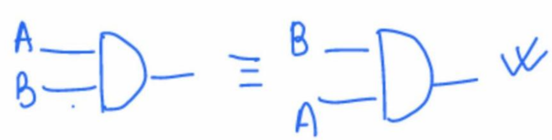
① Idempodent Law :-

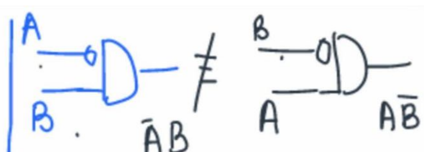
$A\cdot\bar{A}=0$

ⓐ $A=0 \Rightarrow 0\cdot1=0$

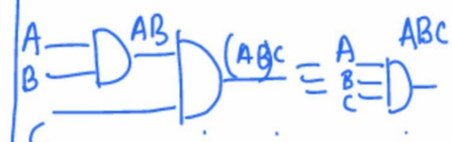
ⓑ $A=1 \Rightarrow 1\cdot0=0$

② Commutative Law :-





③ Associative Law :-

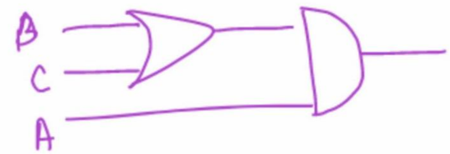


$\equiv (A \odot B) \odot C$

$\neq$

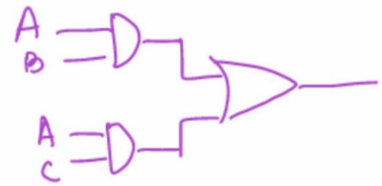
$A \odot B \odot C$

$A(B+C)$



2

$AB + AC$



3

5 Distribution theorem :-

$$(A+B)(A+C) = A + BC$$

6 Transposition theorem:-

$$(A+B)(\bar{A}+C) = AC + \bar{A}B$$

7 Consensus theorem :-

$$AB + AC + \bar{B}C = AB + \bar{B}C$$

#. Some Imp. Point :-

NOT / OR / AND / NAND / NOR → Boolean

OR	
$0+0=0$	$0+0=0$
$0+1=1$	$0+1=1$
$1+0=1$	$1+0=1$
$1+1=1$	$1+1=0$

$1+1=2 \rightarrow 10$

OR → boolean Algebra

X-OR / X-NOR ⇒ Binary gates

Binary algebra

#. $0 \cdot 0 = 0$

$0 \cdot 1 = 0$

$1 \cdot 0 = 0$

$1 \cdot 1 = 1$

$\Rightarrow$ Binary | Boolean

(a) $0 + 0 = 0$

$0 + 1 = 1$

$1 + 0 = 1$

$1 + 1 = 1$

(b) $0 \cdot 0 = 0$

$0 \cdot 1 = 0$

$1 \cdot 0 = 0$

$1 \cdot 1 = 1$

(c) $A + 0 = A$

$A + 1 = 1 \rightarrow 0 + 1 = 1$, $\rightarrow 1 + 1 = 1$

$A + \bar{A} = 1 \rightarrow 0 + 1 = 1$, $\rightarrow 1 + 0 = 1$

$A + A + A + A + \cdots = A$

(d) $A \cdot 0 = 0$

$A \cdot 1 = A$

$A \cdot \bar{A} = 0$

$A \cdot A \cdot A \cdot A \cdots = A$

(8) Law of adjancy :- $(A + B)(\bar{A} + B) = B$

(9) Demorgan's theorem :-

$\overline{A + B + C + D} = \bar{A} \cdot \bar{B} \cdot \bar{C} \cdot \bar{D}$

$\overline{A \cdot B \cdot C \cdot D} = \bar{A} + \bar{B} + \bar{C} + \bar{D}$

Distribution theorem:- POS $\rightleftharpoons$ SOP

$(A + B)(A + C) = A + BC$

$1 + 2 \quad 1 + 3 = 1 + 2\,3$

proof :- $(A + B)(A + C) = A \cdot A + A \cdot C + B \cdot A + BC$

$= A + AC + AB + BC$

$= A(1 + \underbrace{C + B}) + BC = A \cdot 1 + BC = A + BC$

Q. ① $(A+B)(A+\bar{C}) = A+B\bar{C}$

$\quad$ ①+② $\quad$ ①+③ $\quad$ 1+2 3

② $(A+B)(\bar{A}+C) = \times$

③ $(A+B)(A+\bar{B}) = A + \cancel{B\bar{B}}^{\,0} = A+0 = A$ ⇒ Law of adjacency.

$\quad$ ① ② ① ③

④ $AB + A\bar{C} \cancel{=} A+B\bar{C}$

⑤ $A+\bar{A}B = (A+\bar{A})(A+B) = 1\cdot(A+B) = A+B$

$\quad$ ①+②③

$A+\bar{A}B = A+B$ ⇒ Absorbtion theorem

⑥ $A+\bar{A}\bar{B} = A+\bar{B}$ | $A+\bar{A}\bar{B} = (A+\bar{A})(A+\bar{B}) = A+\bar{B}$

$\quad$ ①+②③

⑦ $\bar{A}+AB = \bar{A}+B$ | $\bar{A}+AB = (\bar{A}+A)(\bar{A}+B) = \bar{A}+B$

$\quad$ ① ②③

⑧ $\bar{A}+A\bar{B} = \bar{A}+\bar{B} = \overline{A\cdot B}$

$\bar{A}+A\bar{B} = (\bar{A}+A)(\bar{A}+\bar{B}) = (\bar{A}+\bar{B})$

$\quad$ 1+2 3

Q.

$XY + \overline{XY}\,WZ$

$\quad$ ① ② ③

$= (XY+\overline{XY})(XY+WZ)$

$= \boxed{XY+WZ}$

⑨ $\underbrace{(A+B)(A+\bar{B})}\,\underbrace{(\bar{A}+B)(\bar{A}+\bar{B})}$

$= (A+\cancel{B\bar{B}}^{\,0})(\bar{A}+\cancel{B\bar{B}}^{\,0})$

$= A\cdot\bar{A} = 0$

or

$Y = \overline{\overline{(A+B)\cdot(A+\bar{B})\cdot(\bar{A}+B)\cdot(\bar{A}+\bar{B})}}$

$= \overline{\bar{A}\cdot\bar{B} + \bar{A}\cdot B + A\bar{B} + AB} = \overline{\bar{A}+A} = \bar{1} = 0$

(10) $AB + \bar{A}B + \bar{A}\bar{B} = (A+\bar{A})B + \bar{A}\bar{B}$

$= B + \bar{A}\bar{B} = B + \bar{A}$

or $AB + \bar{A}B + \bar{A}\bar{B} = AB + \bar{A}B + \bar{A}B + \bar{A}\bar{B}$

$= B + \bar{A}$

(11) $F = \bar{A}B + AB + A\bar{B} = B + A$

(12) $A\bar{B}C + ABC + \bar{A}BC$

$= (A\bar{B} + AB + \bar{A}B)C$

$= (A+B)C$ — (1)

$= AC + BC$ — (2)

Basic	NAND	NOR
2 ✓	✗	✓
3 ✗	✓	✗

(13) $\underset{011}{\bar{A}BC} + \underset{101}{A\bar{B}C} + \underset{110}{AB\bar{C}} + \underset{111}{ABC}$

$= \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC + ABC + ABC$

$= BC + AC + AB$

	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$			1	
A		1	1	1

(14) $\bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$ (5:30PM)

$= \bar{A}BC + A\bar{B}C + AB$

$= \bar{A}BC + A(\bar{B}C + B)$

$= \bar{A}BC + A(C+B)$

$= \bar{A}BC + AC + AB$

$= (\bar{A}B + A)C + AB$

$= (B+A)C + AB$

$= BC + AC + AB$ ✓

unacademy

Comprehensive Course on Digital Electronic Circuit for 2021

Function of Function, Dual, Truth Table & Complements

Lesson 2 • June 12, 2020 • Sonu Lal Gupta

$\boxed{\text{Consensus theorem :-}}$

(i) Three varriable must be present
A, B, C

(ii) Combination of two varriable will be in form of SOP or POS

$AB + BC + AC$ or $(A+B)(B+C)(A+C)$

(iii) only one varriable is in complemented and uncomplemented form

$\bar{A}B + AC + \underbrace{BC}_{\text{reductant}}$ or $(A+B)\underbrace{(A+C)}_{\text{reductant}}(\bar{B}+C)$

$AB + \bar{A}C + BC = AB + \bar{A}C$

$(A+B)(A+C)(\bar{B}+C) = (A+B)(\bar{B}+C)$

$\boxed{\text{Proof}}$:-

SOP → logic 1 → A
SOP → logic '0' → $\bar{A}$

POS → logic '1' → $\bar{A}$
POS → logic '0' → A

$F = AB + \bar{A}C + BC$

$= AB\cdot 1 + \bar{A}\cdot 1\cdot C + 1\cdot BC$

$= AB(C+\bar{C}) + \bar{A}(B+\bar{B})C + (A+\bar{A})BC$

$= ABC + AB\bar{C} + \bar{A}BC + \bar{A}\bar{B}C + ABC + \bar{A}BC$

$= ABC + AB\bar{C} + \bar{A}BC + \bar{A}\bar{B}C$ ⇒ standard/canonical form of SOP

$= AB + \bar{A}C$

$= \Sigma m(1, 3, 6, 7)$

$F = AB + \bar{A}C + BC$

	A	B	C	
AB	1	1	0	= 6
	1	1	1	= 7
$\bar{A}C$	0	0	1	= 1
	0	1	1	= 3
BC	0	1	1	= 3
	1	1	1	= 7

$= \Sigma m(1,3,6,7)$

$= m_1 + m_3 + m_6 + m_7$

$= \underbrace{\bar{A}\bar{B}C + \bar{A}BC} + \underbrace{AB\bar{C} + ABC}$

$= \bar{A}C + AB$

	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	0	1 (1)	1 (3)	2
A	4	5	1 (7)	1 (6)

$\bar{A}C$ BC AB

$BC \Downarrow$ Reductant

$Y = \bar{A}C + AB$

F = (A+B)(Ā+C)(B+C)

	A	B	C	
$A+B$	0	0	0	= 0
	0	0	1	= 1
$\bar{A}+C$	1	0	0	= 4
	1	1	0	= 6
$B+C$	0	0	0	= 0
	1	0	0	= 4

$F = \Pi M(0,1,4,6)$

$= M_0 \cdot M_1 \cdot M_4 \cdot M_6$

$= (A+B+C)(A+B+\bar{C})$

$(\bar{A}+B+C)(\bar{A}+\bar{B}+C)$

$= (A+B+\cancel{C\cdot\bar{C}})$

$(\bar{A}+C+\cancel{B\cdot\bar{B}})$

$= (A+B)(\bar{A}+C)$

	$B+C$	$B+\bar{C}$	$\bar{B}+\bar{C}$	$\bar{B}+C$
A	0	0		
$\bar{A}$	0			0

$Y = (A+B)\cdot(\bar{A}+C)$

1. (a) $Y = AB + \bar{A}C + BC = AB + \bar{A}C$

(b) $Y = AB + AC + B\bar{C} = AC + BC$

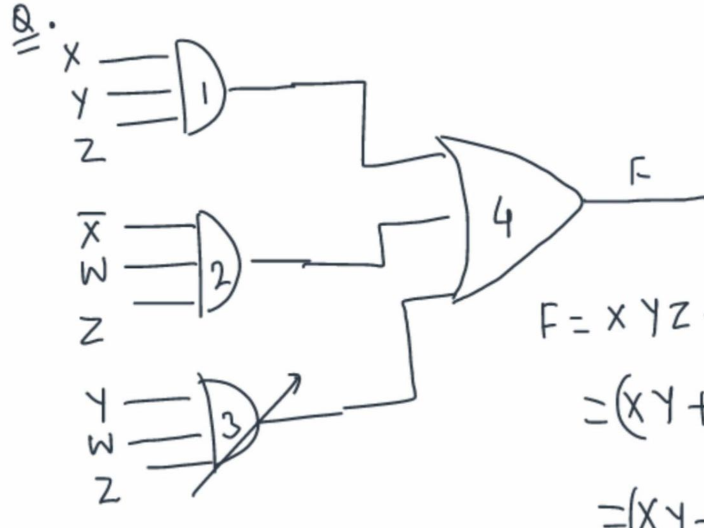
(c) $Y = AB + AC + \bar{B}C = AB + \bar{B}C$

(d) $Y = \bar{A}\bar{B} + \bar{A}C + B\bar{C} = \bar{A}C + B\bar{C}$

(e) $Y = \bar{A}\bar{B} + \bar{A}\bar{C} + B\bar{C} = \bar{A}\bar{B} + B\bar{C}$

(f) $Y = \bar{A}B + A\bar{C} + BC$ ✗

Q.



$F = XYZ + \bar{X}WZ + YWZ$

$= (XY + \bar{X}W + YW)Z$

$= (XY + \bar{X}W)Z$

#. $\boxed{\text{Transposition theorem}}$:–

(a) $(A+B)(\bar{A}+C) = AC + \bar{A}B$

(b) $(A+B)(\bar{A}+\bar{B}) = A\bar{B} + \bar{A}B$

(c) $(\bar{A}+B)(A+\bar{B}) = \bar{A}\bar{B} + AB$

(d) $(B+C)(A+\bar{B}) = AC + \cancel{B\cdot\bar{B}} \to 0 = AC$

$(B+C)(\bar{B}+A)$

$= AB + \bar{B}C$

2. (a) $(A+B)(\bar{A}+C)\boxed{(B+C)}$

$= (A+B)(\bar{A}+C)$

$= AC + \bar{A}B$

(b) $(\bar{A}+\bar{B})(\bar{A}+C)(\bar{B}+\bar{C})$

$= (\bar{A}+C)(\bar{B}+\bar{C}) = \bar{A}\bar{C} + \bar{B}C$