MADE EASY

Mechanical Engineering

Toppers Handwritten Notes

Engineering Mechanics

By- Mudit Raj Sir

ENGINEERING – MECHANICS

GATE SYLLABUS

- Free body diagrams and Equilibrium
- Trusses and Frames
- Virtual Work

- Kinematics and Dynamics of particles & Rigid bodies in plane motion
- Impulse and Momentum (Linear and angular) and Energy Formulations ; Collisions
- Friction in Belt-Pulley, Clutch & Brakes, Screw Jack
- SHM & Vibrations (Civil Engineering)
- Lagrange's Equation

Part(I) (STATICS)

- Equilibrium of Forces
- Friction (statics)
- Truss & Frames
- Virtual Work
- COG/COM/Centroides
- Area & Mass MOI

Part(II) (DYNAMICS) (NEW SYLLABUS)

- Pure Translation Motion
- Work, Energy & Power
- Impact & Collision
- Circular Motion
- Rotational Motion
- Friction in Belt-Pulley
- Friction in Screw Jack
- Friction in Clutch & Brakes
- General Plane Motion
- Rolling Friction
- SHM & Vibrations
- Lagrange's Equation

REFERENCE BOOKS:

1- S.S. Bhavikatti ★★★ (Basic)
2- A. Nelson ★★★
3- A.K. Tayal ★★★ (IAS)
4- Dr. U.C Jindal ★★
5- S. Timoshenko
6- D.H. Young
7- Sukumar Pati
8- J.V. Rao

- GATE →
  - ME
    - Direct → 5-8 Marks (12 Marks) ★★★
    - Indirect → 12-15 Marks
  - CE
    - Direct → 4-6 Marks
    - Indirect → 10-12 Marks

- ESE (Pre.)
  - ME → 2017 → 7 Questions (5-8 Questions)
  - CE → Not added directly.
- CSE (Mains) → (ME+CE) → (35-45) Marks

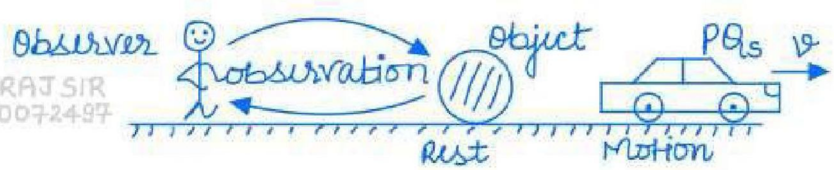
ENGINEERING MECHANICS

(CHAPTER/TOPIC)	(GATE-PYQs)
① Static Equilibrium →	(10-12) Questions
② Friction →	(18-20) Questions
③ Truss →	(8-10) Questions
④ Virtual Work →	(3-4) Questions
⑤ M.O.I. →	(2-3) Questions
⑥ Rectilinear Motion →	(14-16) Questions
⑦ Belt & Pulleys →	(8-9) Questions
⑧ Work, Energy & Power →	(3-4) Questions
⑨ Impact & Collision →	(14-16) Questions
⑩ Circular Motion →	(2-3) Questions
⑪ Rotational Motion →	(12-14) Questions
⑫ Rolling Motion →	(10-11) Questions

ENGINEERING MECHANICS

Definition: It is the study of state of the rigid body; under the application of external forces.

Study: It is the process of observation by an observer about an object/system with respect to a frame of reference.



Physical Quantities: These are the physical characteristics of the body which defines the state of the body.

(a) Fundamental PQs: M, L, T

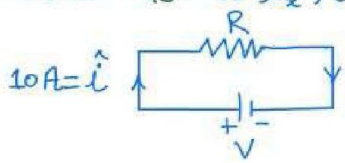
(b) Derived PQs: A, V, ϑ, a, F, p

(c) Scalar PQs: m, A, work → Have only magnitude

(d) Vector PQs: $\vec{W}, \vec{F_x}, \vec{v_x}, \vec{p}, \vec{H}$ → (1) Magnitude

(2) one-direction

** (3) Must follow the vector-laws



** stress $\sigma_{xx}, \sigma_{xy}, \sigma_{yy}, \tau_{xy}$ → 2nd order Tensor Quantities

** M.O.I. I_{xx}, I_{yy}, I_{xy} → Scalar, Tensor etc

** stress $\sigma_{xx}, \sigma_{xy}, \sigma_{yy}, \tau_{xy}$ → 2nd order Tensor Quantities

** M.O.I. I_{xx}, I_{yy}, I_{xy} → Scalar, Tensor etc

Tensor: This is broader term, scalar and vector can be defined as special cases of tensor as,

(1) Zero-order/zero-Rank Tensor → Magnitude + zero-direction (↗ scalar)

(2) First-order/first-Rank Tensor → Magnitude + 1-direction (↗ vector)

(3) Second-order/Second-Rank Tensor → Magnitude + 2-direction (↗ Tensor)

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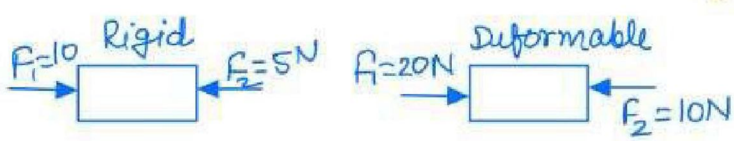
State: It is the condition of the body.

- State
  - Rest (Equilibrium) → (I) Statics
  - Motion (Non-equilibrium) → (II) Dynamics
    - Pure Translation (T)
    - Pure Rotation (R)
    - General Motion (T+R)

Rigid Body: Every body is made up of matter. Matter is some thing which occupies some space and have mass.

- Matter
  - Solid } Solid
    - (1) Rigid (Ideal) → (I) Engg. Mecha.
    - (2) Deformable (Real) → (II) M.O.S/S.O.M.
  - Liquid + Gas } Fluid
    - (3) Incompressible (Ideal) → M.O.F./F.M.
    - (4) Compressible (Real) → Rheology



→ Rest
→ Motion
→ Deformation

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* VECTORS *

* Representation of vectors can be done by two methods

(1) Graphical Method (2) Analytical Method

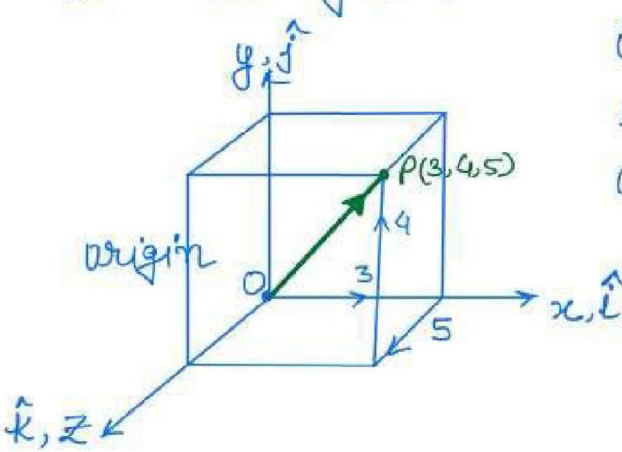
* **Graphical Method:** Vectors can be represented by arrows.

Length = Magnitude

Orientation = Direction

* For complete study a reference system must be there

* **Cartisian Coordinate System:** Assume whole world is 3-D coordinate system.

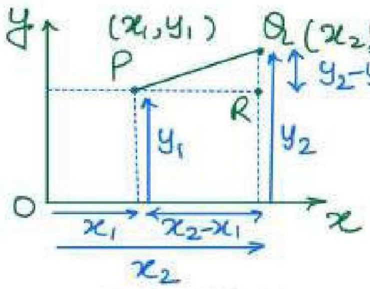


$\vec{OP} = \vec{P} = 3\hat{i} + 4\hat{j} + 5\hat{k}$ —— (1)

length = mag. = $|\vec{OP}|$

direction = angles made from x, y + z axis —— (2)

Distance between any two points:



for 2-D case:

$$PQ^2 = PR^2 + QR^2$$

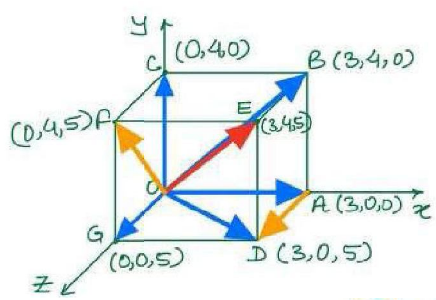
$$PQ^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{—— (1)}$$

Similarly for 3-D: $P(x_1, y_1, z_1)$, $Q(x_2, y_2, z_2)$

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \quad \text{—— (2)}$$

Vectors in 3-D system:



$\vec{OA} = 3\hat{i} + 0\hat{j} + 0\hat{k} = 3\hat{i} = \text{1-D}$

$\vec{OC} = 0\hat{i} + 4\hat{j} + 0\hat{k} = 4\hat{j} = \text{1-D}$

$\vec{OG} = 0\hat{i} + 0\hat{j} + 5\hat{k} = 5\hat{k} = \text{1-D}$

$\vec{OD} = 3\hat{i} + 0\hat{j} + 5\hat{k} = 3\hat{i} + 5\hat{k} = \text{2-D}$

$\vec{OB} = 3\hat{i} + 4\hat{j} + 0\hat{k} = 3\hat{i} + 4\hat{j} = \text{2-D}$

$\vec{OE} = 3\hat{i} + 4\hat{j} + 5\hat{k} = \text{3-D}$

$\vec{AD} = (3-3)\hat{i} + (0-0)\hat{j} + (5-0)\hat{k} = 5\hat{k} = \text{1 D}$

$\vec{OF} = (0-0)\hat{i} + (4-0)\hat{j} + (5-0)\hat{k} = 4\hat{j} + 5\hat{k} = \text{2-D}$

In general:

$\vec{A} = x\hat{i} + y\hat{j} + z\hat{k}$ —— (1) (3-D)

$\vec{A} = x\hat{i} + y\hat{j}$ —— (2) (2-D)

$\vec{A} = x\hat{i}, y\hat{j}, z\hat{k}$ —— (3) (1-D)

Analytical Approach: If $\vec{A} = \vec{OA} = x\hat{i} + y\hat{j} + z\hat{k}$ —— ①

Magnitude: $|\vec{A}| = |\vec{OA}| = A = \sqrt{x^2 + y^2 + z^2}$ —— ②

* $\vec{A} = A \cdot \hat{A}$ —— ③

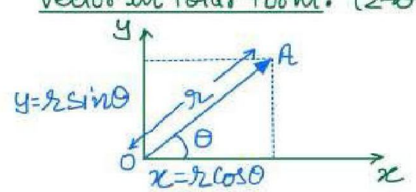
A → magnitude, $\hat{A}$ → unit vector

$$\therefore \hat{A} = \frac{\vec{A}}{A} = \frac{x\hat{i} + y\hat{j} + z\hat{k}}{\sqrt{x^2+y^2+z^2}} = \left(\frac{x}{\sqrt{x^2+y^2+z^2}}\right)\hat{i} + \left(\frac{y}{\sqrt{x^2+y^2+z^2}}\right)\hat{j} + \left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)\hat{k} \quad \text{—— ④}$$

** The vector having unit magnitude (1) called unit vector.

** Unit vector of any vector defines its direction.

Vector in Polar Form: (2-D case)



$\vec{OA} = x\hat{i} + y\hat{j}$ ★★★

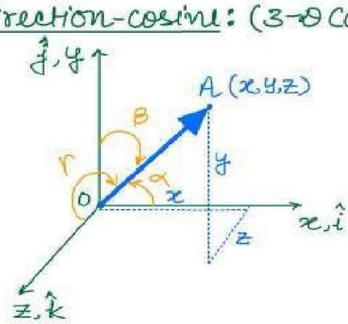
$$\boxed{\vec{OA} = r\cos\theta\,\hat{i} + r\sin\theta\,\hat{j}} \quad \text{—— ①}$$

$|\vec{OA}| = \sqrt{(r\cos\theta)^2 + (r\sin\theta)^2}$

$|\vec{OA}| = \sqrt{r^2}$

$|\vec{OA}| = r$

Direction-cosine: (3-D Case)



$\vec{OA} = x\hat{i} + y\hat{j} + z\hat{k}$ —— ①

Let, $|\vec{OA}| = r = \sqrt{x^2+y^2+z^2}$ —— ②

$x = r\cos\alpha \rightarrow \cos\alpha = \frac{x}{\sqrt{x^2+y^2+z^2}}$

$y = r\cos\beta \rightarrow \cos\beta = \frac{y}{\sqrt{x^2+y^2+z^2}}$

$z = r\cos\gamma \rightarrow \cos\gamma = \frac{z}{\sqrt{x^2+y^2+z^2}}$ —— ③

*** Here, $\cos\alpha$, $\cos\beta$ & $\cos\gamma$ called direction cosines of the vector.

*** From ③ → $\boxed{\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1}$ — ④

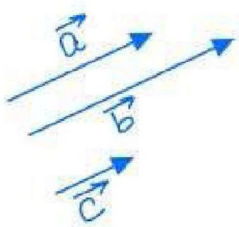
*** Hence → $(1-\sin^2\alpha) + (1-\sin^2\beta) + (1-\sin^2\gamma) = 1$

$3 - (\sin^2\alpha + \sin^2\beta + \sin^2\gamma) = 1$

$\therefore \boxed{\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2}$ — ⑤

Types of Vectors:

① Parallel vectors: Having same direction but may be diffe. in magnitude.



Let $|\vec{b}| = 8$, $|\vec{a}| = 4$ } $|\vec{b}| = 2|\vec{a}|$

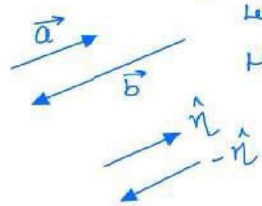
$\boxed{b = 2a}$ — ①

* $\vec{b} = b\cdot\hat{b} = b\cdot\hat{\eta}$
$\vec{a} = a\cdot\hat{a} = a\cdot\hat{\eta}$ } $\hat{a} = \hat{b} = \hat{\eta}$

* $\vec{b} = (2)a\cdot\hat{\eta} = 2\vec{a}$ — ②

* In general $\boxed{\vec{b} = m\cdot\vec{a}}$ — ③ m = (scalar or constant)

② **Antiparallel Vector:** When directions are just opposite, and magnitude may or may not be equal.



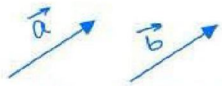
Let $|\vec{b}| = 2|\vec{a}|$ —①

Here $\vec{b} = -2\vec{a}$ —②

* In general $\boxed{\vec{b} = -m\vec{a}}$ —③

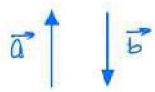
(m = (+)ve constant)

③ **Equal vectors:** when both magnitude + direction are same.



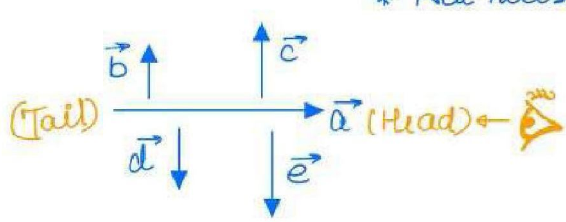
$\boxed{\vec{a} = \vec{b}}$ —①

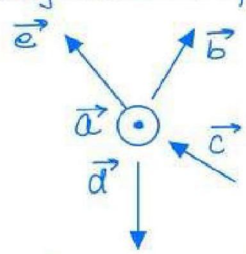
④ **Opposite vectors:** When magnitude is same but direction is opposite called opposite vectors.



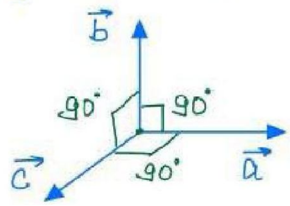
$\boxed{\vec{a} = -\vec{b}}$ or $\boxed{\vec{b} = -\vec{a}}$ —②

⑤ **Normal vectors:** When directions are mutually perpendicular, may or may not equal in magnitude.
* Not necessarily in same plane.





⑥ **Orthonormal Vectors:** When any 3-vectors are mutually perpendicular to each other.



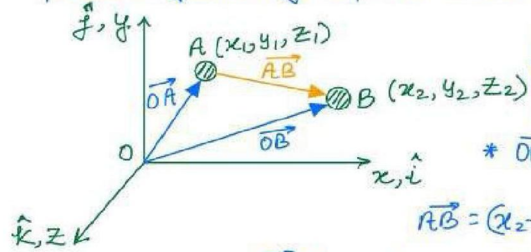
⑦ Null/zero vector: A vector with zero magnitude and having no-specific direction called zero-vector.

$$\vec{a} = a \cdot \hat{a} \quad \text{—(1)}$$

a ↓ zero; $\hat{a}$ → No-specific direction

* As both head and tail will come at same point so direction can not be predicted/determine.

** ⑧ Position-Vector: Any vector connecting the body at any position from origin point called 'Position-vector'.



$$\vec{OA} = \vec{A} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k} \quad \text{—(1)}$$

$$\vec{OB} = \vec{B} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k} \quad \text{—(2)}$$

* $\vec{OA}$ + $\vec{OB}$ are position vectors.

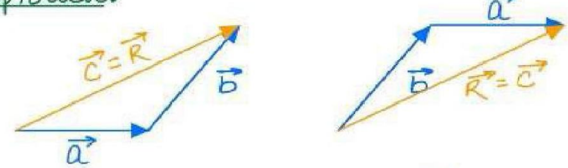
$$\vec{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k} \quad \text{—(3)}$$

* $\vec{AB}$ is not a position vector.

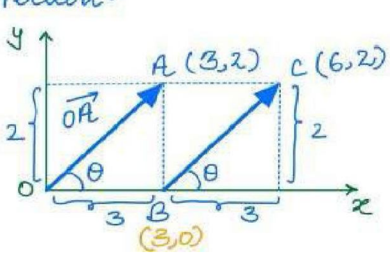
* Addition of Vectors *

* When any two vectors are added their resultant is obtained.

Graphical Approach:



** Translational Property of vectors: "Vectors can be translated in the space without changing their mag. + direction".



$$\vec{OA} = (3-0)\hat{i} + (2-0)\hat{j}$$

$$\vec{OA} = 3\hat{i} + 2\hat{j} \quad \text{—(1)}$$

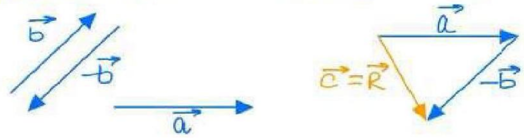
$$\vec{BC} = (6-3)\hat{i} + (2-0)\hat{j}$$

$$\vec{BC} = 3\hat{i} + 2\hat{j} \quad \text{—(2)}$$

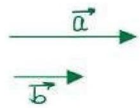
* Hence, $\vec{OA} = \vec{BC}$

Substraction of vectors: It can be done by addition of opposite vector.

* $\vec{C}$ or $\vec{R} = \vec{a} + (-\vec{b})$

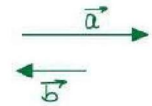


Ex→



Draw $\vec{a}+\vec{b}$, $\vec{a}-\vec{b}$

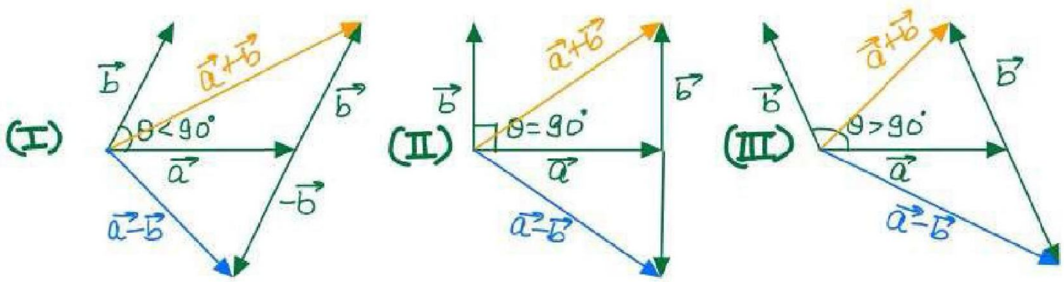
Ex→



Draw $\vec{a}+\vec{b}$, $\vec{a}-\vec{b}$

** In vectors everything is possible like,

$\vec{a}+\vec{b} > \vec{a}-\vec{b}$, $\vec{a}+\vec{b} < \vec{a}-\vec{b}$, $\vec{a}+\vec{b} = \vec{a}-\vec{b}$



$|\vec{a}+\vec{b}| > |\vec{a}-\vec{b}|$ — $\theta<90°$ (Acute angle)

$|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$ — $(\theta=90°)$

$|\vec{a}+\vec{b}| < |\vec{a}-\vec{b}|$ — $\theta>90°$ (Obtuse)