MADE EASY

Civil Engineering

Toppers Handwritten Notes

OPENCHANAL FLOW

By-Badal Sony Sir

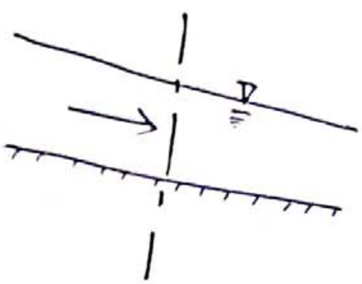
Gate : 2-4 marks.
ESE : OBJ → 5-6 Q
CONV → 50-60 marks

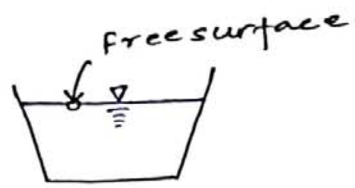
1. Introduction :
2. Uniform flow
3. Energy Depth Relationship.
4. Gradually varied flow.
5. Rapidly Varied flow → Hydraulic Jump
6. Surges

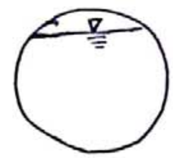
Introduction :

→ Open channel flow refers to the flow of liquid in channel open to atmosphere or in a partially filled conduit.

→ It is characterised by the presence of liquid-gas interface called free surface.







partially filled conduit.

NOTE : • The driving force in an open channel flow is gravity.

• Shear stress on the free surface is zero.

Types of channels :

(i) Prismatic and Non-prismatic channel:

If cross-section, shape, size, bed slope remains constant in the direction of flow then the channel is called prismatic otherwise non-prismatic.

(ii) Rigid and Mobile Boundary channel :

A. Rigid Boundary channel : Only depth varies with space and time.

Boundaries not deformable.

→ Shape and roughness parameter are not function of flow.

eg: lined canal and sewers.

b. Mobile Boundary channel:

→ In this case the depth, width, bed slope as well as layout are functions of space and time.

eg: Unlined canals.

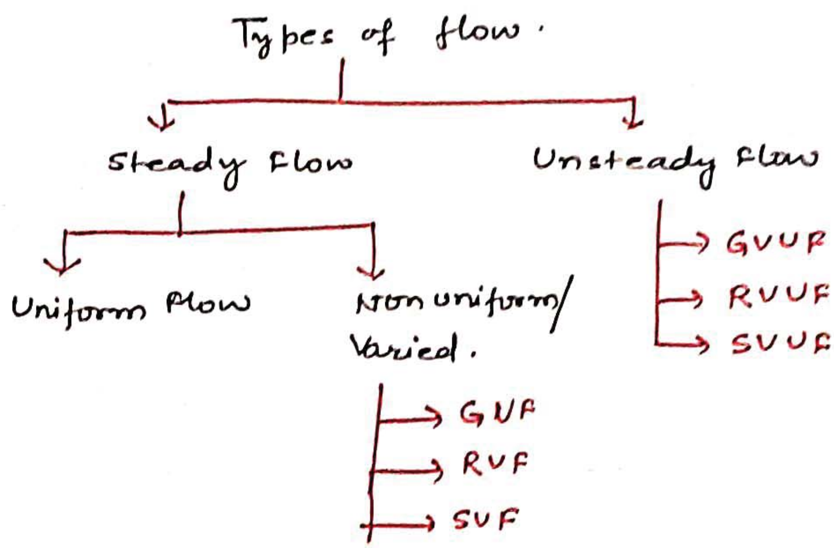
NOTE: The rigid boundary channel has one degree of freedom while mobile boundary has four degrees of freedom.

We will study only rigid boundary channels.

Rigid Boundary $\not\rightarrow$ Prismatic

Prismatic ⟶ Rigid Boundary.

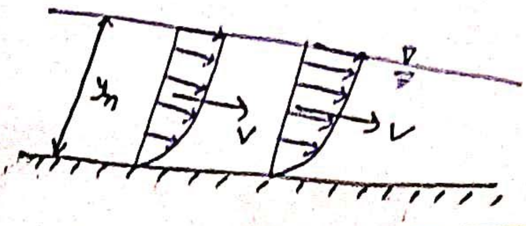
Types of flow:



Uniform Flow:

→ Flow is called steady uniform if the depth of flow does not vary in space.

→ The underlined assumption ~~that~~ is that the velocity also does not vary which means that the cross-section parameter, roughness parameter, slope parameter are not varying.



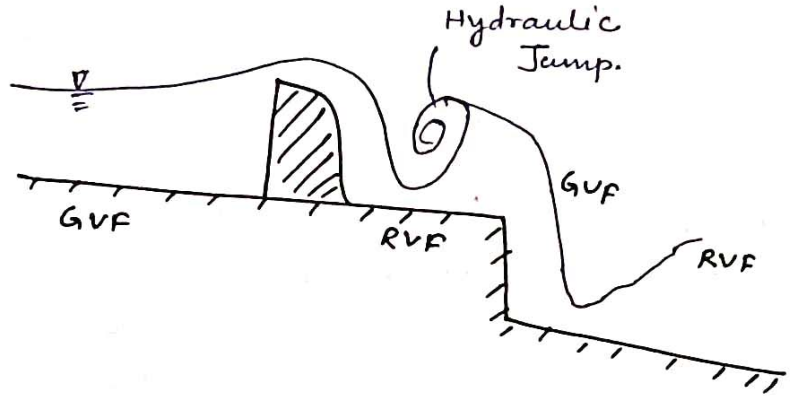
yn = Depth of flow.

V = Avg. velocity of flow.

→ In uniform flow the energy gained due to elevation fall is lost due to flow i.e frictional losses.

→ In prismatic channel, constant depth flow means uniform flow and the depth of flow is called normal depth of flow. (y_n)

2. Non-uniform / Varied flow:



→ Presence of obstruction in channel such as wier, dropping bed, change in slope or cross-section causes the flow to vary, this flow is called non-uniform flow or varied flow.

→ Flow is called gradually varied if the depth changes gradually over a long distance of channel.

→ Curvature of stream line is gentle in this case.

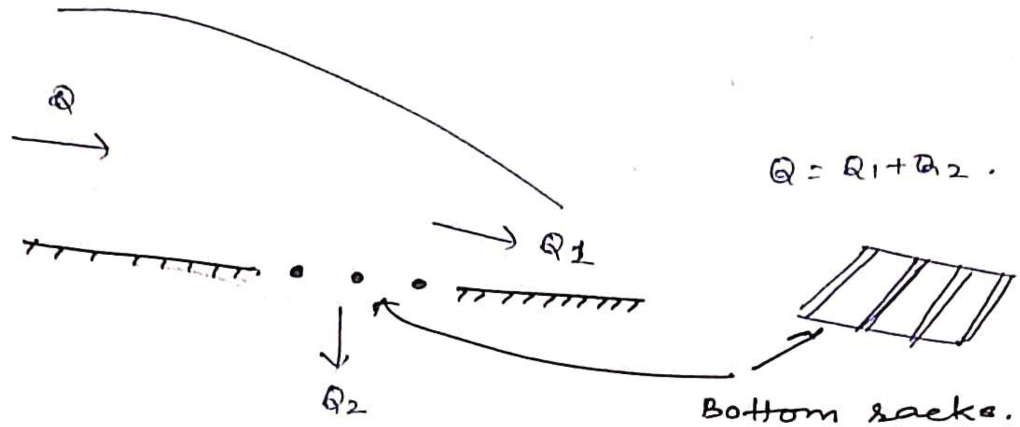
→ If the depth of flow changes significantly over a short distance such that the curvature changes rapidly, the flow is called rapidly varied flow.

eg: Hydraulic Jump.

NOTE: Friction plays an important role ~~but~~ in GVF but not important case of RVF.

If some flow is added or extracted from the system, the flow is called spatially varied flow.

eg: flow over bottom rack.

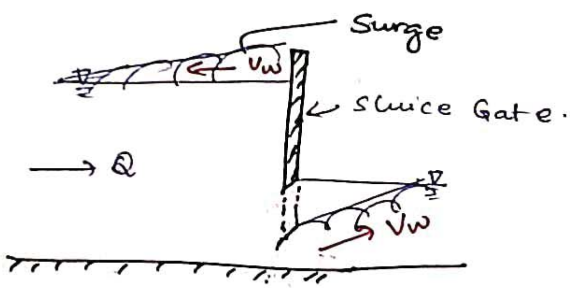


Unsteady flow:

a. Gradually varied Unsteady flow:
   eg: Passage of flood wave in rivers.

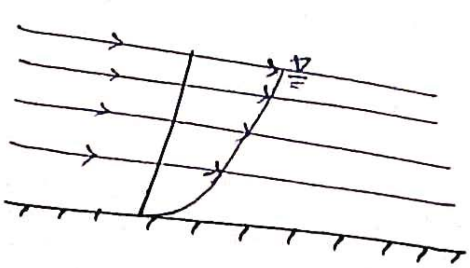
b. Rapidly varied unsteady flow:
   eg: surges, tidal bores, breaking of waves on shore.

c. Spatially Varied Unsteady flow:
   eg: Surface runoff due to rain fall.



Laminar Flow and Turbulent flow:

→ when the flow occurs such that one layer of the liquid slides pas the other as if one lamina is sliding over the other, the flow is called Laminar flow, where there would be <u>no momentum transfer</u> between different layers.



→ However if water from one layer goes into the other and visa-versa, there could be momentum transfer between different layers such a flow is called turbulent flow.

$$Re = \frac{VR}{\nu}$$

Re = Reynold's number. (dimensionless)

V = Avg. velocity.

R = Hydraulic Radius.

$= \frac{A}{P}$ A = Area of x-section. P = wetted perimeter.

ν = Kinematic viscosity. (m^2/s)

$\nu = \frac{\mu}{\rho}$ μ = Dynamic viscosity. (Pa-s)

if $Re < 500$ Laminar flow.

$500 < Re < 2000$ Transition flow.

$Re > 2000$ Turbulent flow.

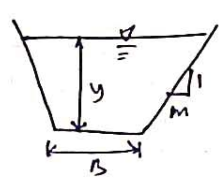
Critical / Subcritical / super-critical flow:

$$Fr = \frac{V}{\sqrt{gA/T}}$$

Fr = Froude's no. (Dimensionless)

A = Area of x-section.

T = Top width.



$T = B + 2my.$

$A = By + my^2$

Critical	Subcritical	Super-critical
$Fr = 1$	$Fr < 1$	$Fr > 1$
$V = V_c$	$V < V_c$	$V > V_c$
$y = y_c$	$y > y_c$	$y < y_c$

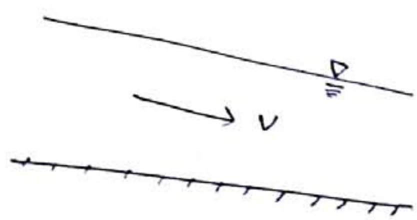
V_c = critical velocity $= \sqrt{gA/T}$ y_c = critical depth.

Celerity (C_0):

Denominator of Froude's no. represents a speed with which disturbance created to flow travels in still water, is called Celerity (C_0).

$$C_0 = \sqrt{gA/T} = \sqrt{g L_c}$$

L_c = characteristic Length.



$$\left(V_{(wave/ground)}\right)_{u/s} = \left(V_{(wave/water)}\right)_{u/s} + \left(V_{(water/ground)}\right)_{u/s}$$

$$\left(V_{wave/ground}\right)_{u/s} = C_0 - V$$

For subcritical flow. ~~For super-critical.~~

$Fr < 1$

$\frac{V}{C_0} < 1$

$C_0 - V > 0$

d/s control.

→ At low flow velocity ($Fr < 1$) a small disturbance to the flow will cause disturbance wave which travels u/s with the velocity $C_0 - V$ wrt a stationary observer.

→ Due to upstream movement of wave, upstream condn gets affected. Thus in case sub-critical flow condition upstream is affected by the condn at downstream. and down stream section is taken as control section.

For super-critical Flow:

$Fr > 1$

$\frac{V}{C_0} > 1$

$C_0 - V < 0$

U/s control.

At high flow velocity $Fr > 1$, the upstream flow velocity of wave $(C_0 - V)$ will become negative ie the disturbance wave will not travel upstream, it will travel downstream with a velocity of $(V - C_0)$

Hence, flow condn downstream will be affected and super critical flow has upstream control.

NOTE: Sub-critical flow has downstream control while super-critical flow has upstream control.

When $Fr = 1$, flow is critical and the disturbance velocity $C_0 - V = 0$ ie distrubance wave will not travel at all.

Q: A wide rect. channel is 1m deep and has a velocity of flow $V = 2.13$ m/s. If the disturbance is caused and elementary wave can travel upstream with a velocity of

a) 1 m/s

b) 3.13 m/s

c) 2.13 m/s

d) 5.26 m/s.

$$C_0 = \sqrt{g\,A/T} = \sqrt{g \times \frac{By}{B}}$$

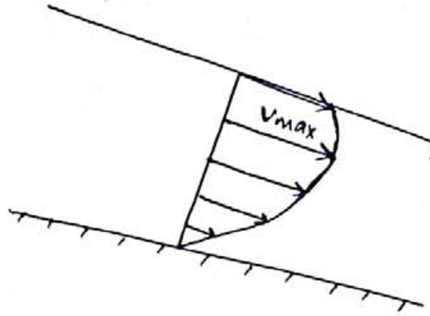
$$= \sqrt{g \cdot y}$$

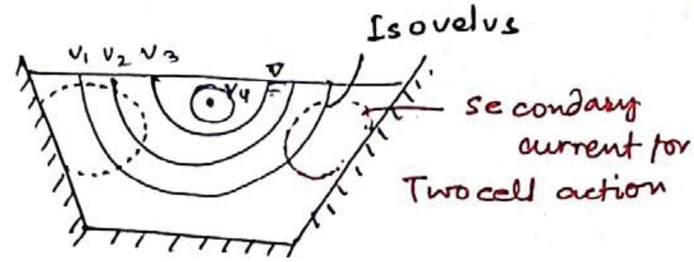
$$C_0 = \sqrt{9.81}$$

3.13 m/s.

$(V_{wave/Ground}) = C_0 - V = 3.13 - 2.13 = 1$ m/s

Velocity Distribution:





$V_4 > V_3 > V_2 > V_1$

Isovelves: contours of equal velocity

Aspect Ratio = $\frac{\text{Depth}}{\text{width}}$.

Reduction or dip in the velocity is because of secondary current which is a function of aspect ratio.

If aspect ratio is large, depth at which maxm velocity occurs is deeper.

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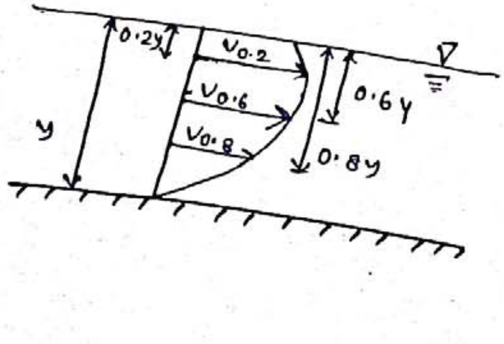
Average Velocity

1. $$\boxed{V_{avg} = \frac{\int v \, dA}{A}}$$

V = Average velocity.

v = Actual velocity.

2. 

$$\boxed{V_{avg} = \frac{V_{0.2} + V_{0.8}}{2}}$$

Or

$\boxed{V_{avg} = V_{0.6}}$ (Less reliable)

3. $\boxed{V_{avg} = K \cdot V_{surface}}$

Where K is a constant varies b/w 0.8 - 0.95.

Correction factors:

1. Kinetic Energy Correction factor (α) or Coriolis Co-efficient.

For an elemental area dA, K.E flux through it is.

$$\frac{\text{Mass}}{\text{time}} \times \frac{\text{KE}}{\text{Mass}} = \rho v dA \times \frac{1}{2} v^2$$

Now for total area "A", K.E. flux is

$$\int \rho v dA \times \frac{v^2}{2} = \alpha \cdot \rho V A \cdot \frac{V^2}{2}$$

$$\therefore \boxed{\alpha = \frac{\int v^3 dA}{V^3 A}}$$

v = Actual velocity
V = Avg. velocity

2. Momentum Correction factor (β) or Boussinesq Co-eff.

for an elemental area dA, Momentum flux through it is,

$$\frac{\text{Mass}}{\text{time}} \times \text{velocity} = \rho v dA \times v$$

Now, for total area "A", momentum flux is.

$$\int \rho v dA \times v = \beta \cdot \rho V A \times V$$

v = Actual velocity
V = Avg. velocity.

$$\boxed{\beta = \frac{\int v^2 dA}{V^2 A}}$$

NOTE: It is usual practice to assume $\alpha = \beta = 1$ when no other specific info about the co-eff. is available.

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:5. $v = k\sqrt{y}$

for a rect. channel.

$dA = B.dy.$

$$V_{avg} = \frac{\int_0^{y_0} k\sqrt{y} \cdot B \cdot dy}{B \cdot Y_0}$$

$$= \frac{1}{3/2} \cdot \frac{K \cdot y_0^{3/2}}{y_0}$$

$$V_{avg.} = \frac{2}{3} K y_0^{1/2}$$

$$\alpha = \frac{\int v^3 dA}{V^3 A}$$

$$= \frac{\int k^3 \cdot y^{3/2} \cdot B \cdot dy}{\left(\frac{2}{3} k y_0^{1/2}\right)^3 \cdot B \cdot y_0}$$

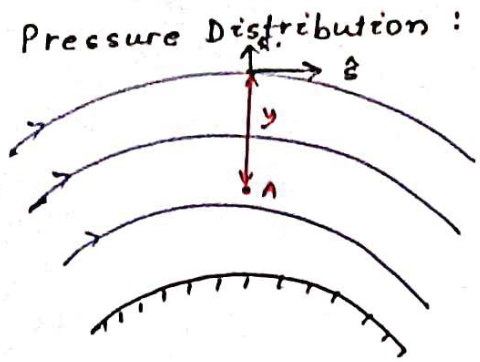
$$= \frac{y_0^{5/2}}{\frac{5}{2} \cdot \frac{8}{27} \cdot y_0^{5/2}} = \frac{27}{20}$$

$$\beta = \frac{\int v^2 dA}{V^2 A}$$

$$= \frac{\int k^2 \cdot y \; dy}{\left(\frac{2}{3} k \cdot y_0^{1/2}\right)^2 \cdot y_0}$$

$$= \frac{\cancel{y_0^2}}{2 \cdot \frac{4}{9} \cdot \cancel{y_0^2}} = \frac{9}{8}$$

Pressure Distribution :



By Euler's formula , we have

$$\frac{-\partial(p+\gamma z)}{\partial n} = \rho a_n$$

where a_n = normal acceleration.

$$a_n = \frac{V^2}{r}$$

⇒ If the streamlines are straight lines, then

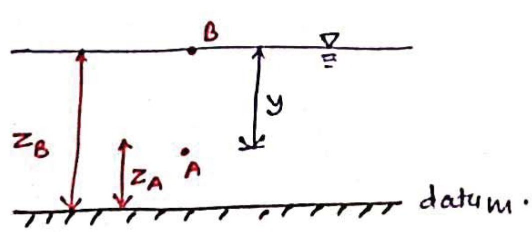
$$r = \infty \Rightarrow a_n = \frac{V^2}{r} \longrightarrow 0$$

$$-\frac{\partial(p+\gamma z)}{\partial n} = 0$$

$$p+\gamma z = \text{constant.}$$

$$\frac{p}{\gamma} + z = \text{constant.}$$

∴ $\boxed{\text{Piezometric head = constant.}}$



$$p_A + \gamma z_A = \cancel{p_B}^{0} + \gamma \cdot z_B .$$

$$p_A = \gamma(z_B - z_A)$$

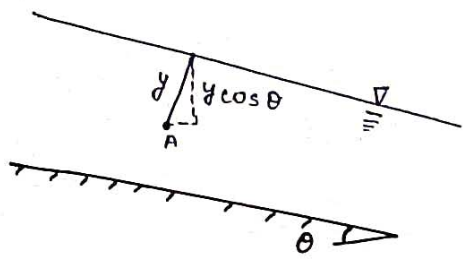
$$\boxed{p_A = \gamma y = \rho g y}$$

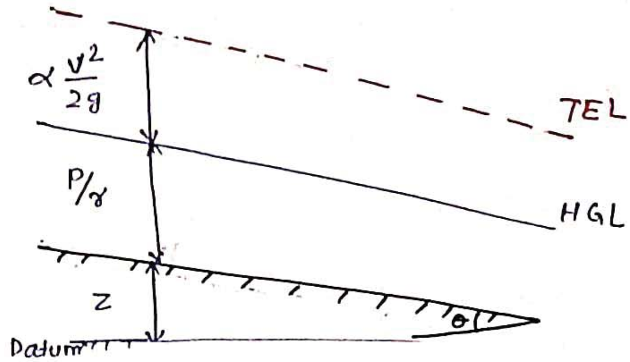
ie pressure distribution is hydrostatic.

Non-curvilinear flow:

TEL : Total Energy Line

HGL : Hydraulic Gradient line.





$P_A = \gamma y \cos\theta$

If θ is small,

$P_A = \gamma y$

→ If $\left(\frac{P}{\gamma} + z\right)$ is plotted in the direction of flow we get HGL.

→ If $\left(\frac{P}{\gamma} + z + \alpha\frac{V^2}{2g}\right)$ is plotted in the direction of flow, we get TEL

→ For smaller slope, HGL will coincide with the free surface and for large slope HGL will be below the free surface. ($y\cos\theta < y$)

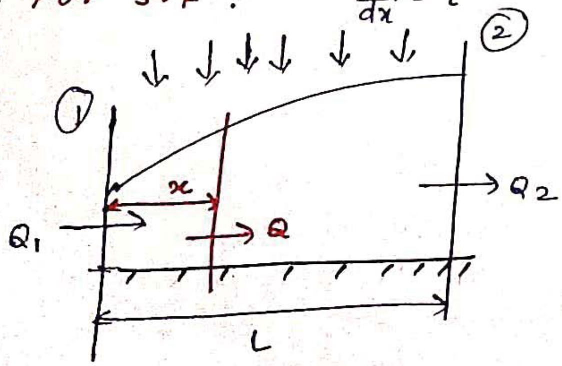
Continuity Equation:

a. For steady flow (Uniform, GVF, RVF)

$$Q = A_1V_1 = A_2V_2$$

ie $\frac{dQ}{dx} = 0$.

b. For SVF : $\frac{dQ}{dx} = q$



$$Q = Q_1 + \int_0^x q\,dx$$

Rate of add^n of discharge $\left(\frac{dQ}{dx}\right) = q$

Now at distance x from sec 1, the discharge is given by.

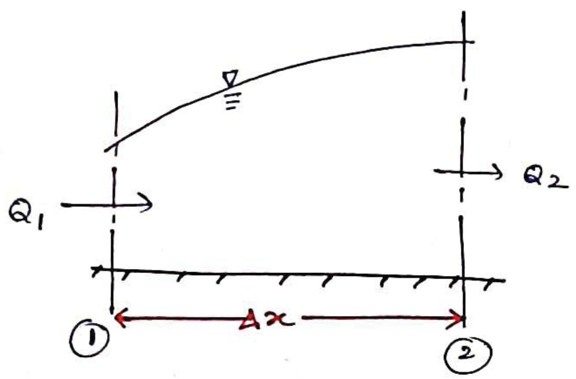
$$Q = Q_1 + \int_0^x q\,dx.$$

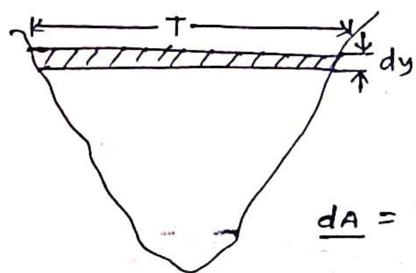
If q = constant

then $Q_{at\,x} = Q_1 + qx$.

$Q_{at\,L} = Q_1 + qL.$

C. Unsteady Flow (GVUF, RVUF) :





$dA = T\,dy$

$\frac{dA}{dy} = T$

$Q_2 \rightarrow$ downstream discharge

$Q_1 \rightarrow$ Upstream discharge.

if $Q_2 > Q_1$

Net discharge going out of the boundaries in "Δt" time = depletion in storage within it.

$$\frac{\partial Q}{\partial x} \times \Delta x \times \Delta t = -\frac{\partial A}{\partial t}\Delta t \times \Delta x$$

$$\Rightarrow \left(\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial y} \times \frac{\partial y}{\partial t}\right)\Delta x\,\Delta t = 0 \qquad \left(\frac{\partial A}{\partial y} = T\right)$$

$$\Rightarrow \boxed{\frac{\partial Q}{\partial x} + T\frac{\partial y}{\partial t} = 0}$$

d. For SVUF

$$\frac{\partial Q}{\partial x} + T\frac{\partial y}{\partial t} = \pm q$$

(+) sign for addition

(−) sign for extraction.

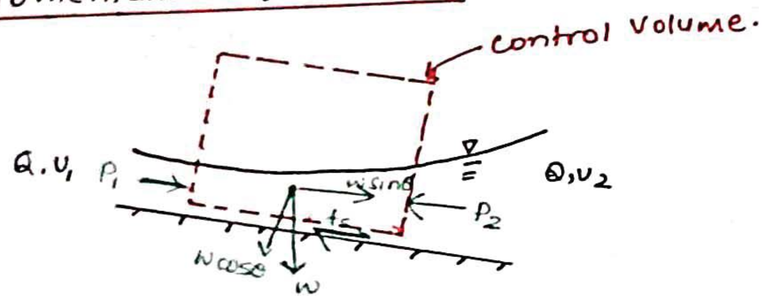
Q: $T = 20m$.

$Q_2 = 10\ m^3/s$.

$$\frac{(10 - Q_1)}{2000} + \frac{20 \times 0.2}{60 \times 60} = 0$$

$Q_1 = 12.22 m^3/s$

Momentum Equation: (only for steady flow)



Net force acting on CV in the direction of flow. $= P_1 - P_2 - F_s + W\sin\theta$ ——— (i)

P_1, P_2 are pressure force.

$P = \gamma \bar{y} A$ $\quad \bar{y}$ = depth of the CG of area from free surface.

A = Area of cross-section.

Rate of change of Momentum of CV. $= M_2 - M_1$ ——— (ii)

M_1 and M_2 → Momentum flux.

$M = \rho Q V$

$M_1 = \rho Q V_1 \qquad M_2 = \rho Q V_2$

From (i) and (ii)

$$P_1 - P_2 + W\sin\theta - F_s = M_2 - M_1$$

$$P_1 = \gamma \bar{y}_1 A_1 \quad \text{and} \quad P_2 = \gamma \bar{y}_2 A_2$$

∴ in actual form, for steady flow, the momentum eqn is written as,

Net force acting on CP in direction of flow = Moment flux going out of the CV in x-direction − Momentum flux coming in to the CV in x-direction.

Specific Force :

$$F_{sp} = \frac{P+M}{\gamma}$$

P = Pressure Force

M = Momentum Flux.

$P = \gamma \bar{y} A$.

$M = \rho Q V$

By momentum equation.

$$P_1 + W\sin\theta - P_2 - F_s = M_2 - M_1$$

For frictionless horizontal channel.

$\sin\theta = 0$, $F_s = 0$

$$P_1 - P_2 = M_2 - M_1$$

$$\frac{P_1 + M_1}{\gamma} = \frac{P_2 + M_2}{\gamma}$$

$\Rightarrow \boxed{F_{sp} = \text{constant}}$

Thus specific force is constant for a horizontal frictionless channel.