MADE EASY

Civil Engineering

Toppers Handwritten Notes

Engineering Mechanics

By-Aakash Tiyagi Sir

VECTOR CALCULUS

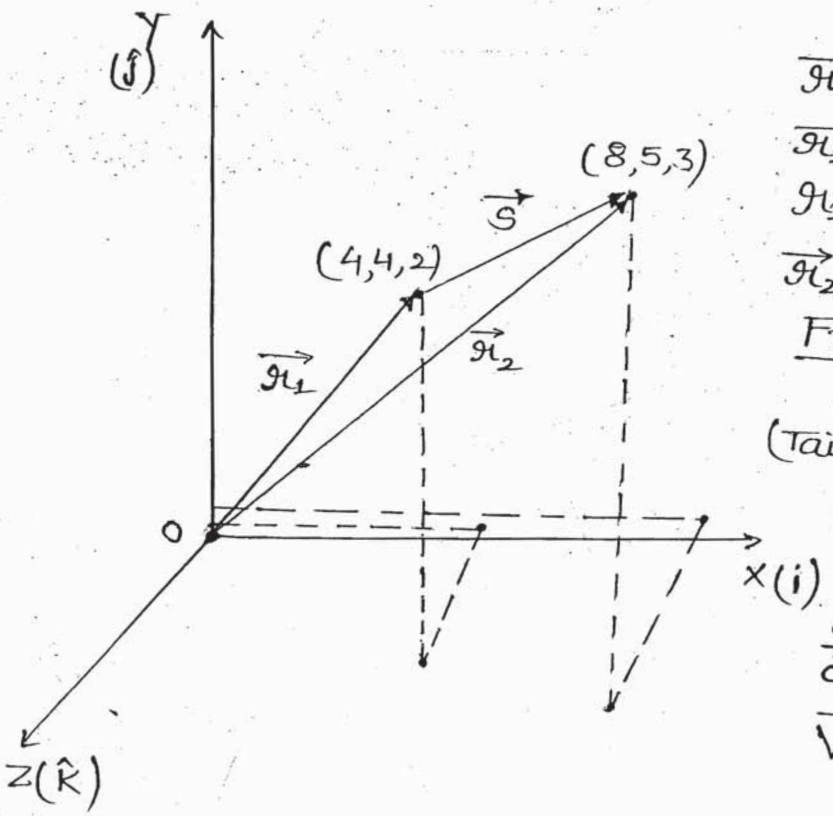
origin pt. : Pt. wch is fix in space.

$\vec{r}$: Position Vector.

Vector : only when it should follow Δ^{le} law of add^n.

r : Magnitude

$\vec{S}$: Displacement Vector or Change in position vector



$$\vec{r_1} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$$

$$\vec{r_1} = (4\hat{i} + 4\hat{j} + 2\hat{k})$$

$$r_1 = \sqrt{4^2 + 4^2 + 2^2}$$

$$\vec{r_2} = (8\hat{i} + 5\hat{j} + 3\hat{k})$$

From Δ Law :

$$\vec{r_1} + \vec{S} = \vec{r_2}$$

(Tail to head add) (head to head resultant)

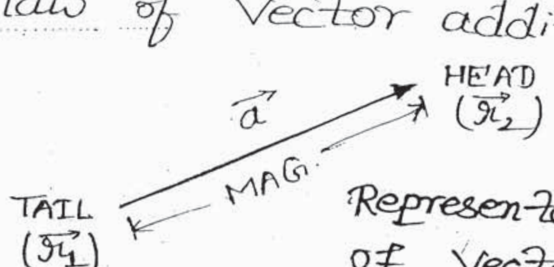
$$\vec{S} = \vec{r_2} - \vec{r_1} = 4\hat{i} + \hat{j} + \hat{k}$$

$$\frac{dS}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$$\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$$

Vector is the physical quantity wch is having both Magnitude as well as dir^n & it should obeys Δ^{le} law of Vector addition.

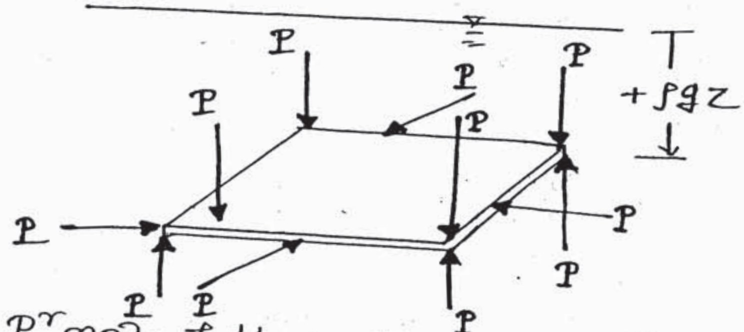
NOTE: Some physical quantity have both magnitude as well as dir^n but they r not obeying Δ^{le} law of Vector addition (Ex: Pressure, Current)

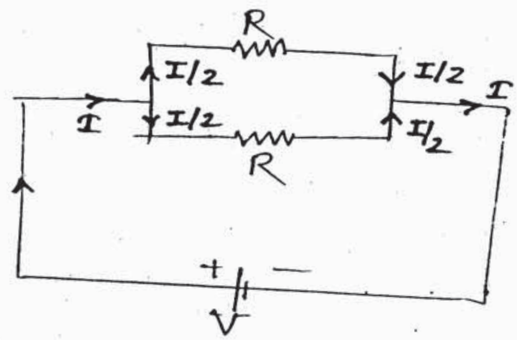


Representation of Vector

EX: PRESSURE → Scalar Quantity → External

CURRENT





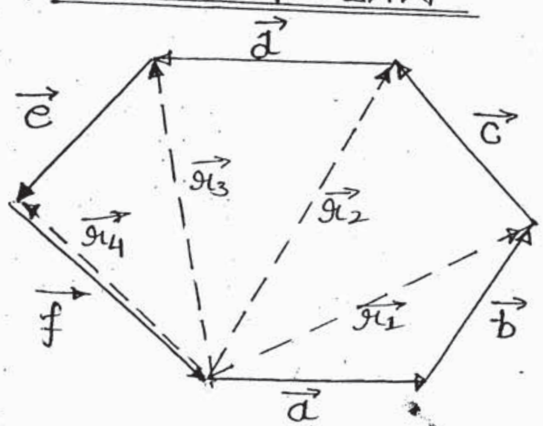
P^r not follow Δ^{le} law so it is not cancel each other components, so on the surface always acts P^r (P).

Current also not a vector quantity. ∴ $I/2$ & $I/2$ not cancel each other & they make by add^n current (I)

* STRESS → Tensor quantity → Internal

LAWS OF VECTOR

1.) POLYGON LAW



$$\vec{r_1} = \vec{a}_1 + \vec{b}_1$$

$$\vec{r_2} = \vec{r_1} + \vec{c} = (\vec{a} + \vec{b}) + \vec{c}$$

$$\vec{r_3} = \vec{a} + \vec{b} + \vec{c} + \vec{d}$$

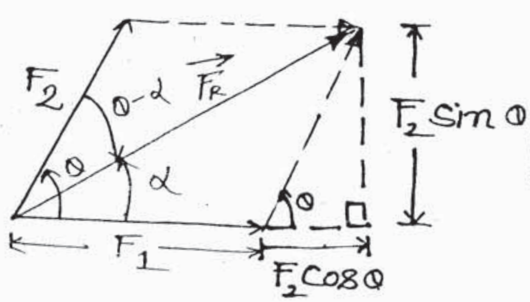
$$\vec{r_4} = \vec{a} + \vec{b} + \vec{c} + \vec{d} + \vec{e} \quad \text{—(1.)}$$

$$0 = \vec{a} + \vec{b} + \vec{c} + \vec{d} + \vec{e} + f \quad \text{—(2.)}$$

$$\vec{r_4} = -f$$

"Negative Vector"

2.) PARALLOGRAM LAW



$$F_R = \sqrt{(F_1 + F_2 \cos θ)^2 + (F_2 \cos θ)^2}$$

Magnitude: *

$$\boxed{F_R = \sqrt{F_1^2 + F_2^2 + 2 F_1 F_2 \cos θ}}$$

Direction: *

$$\boxed{\tan θ = \frac{F_2 \sin θ}{(F_1 + F_2 \cos θ)}}$$

C.S. 2006

Q.1 When 2 forces F_1 & F_2 mutually at right angle then their resultant is 10 KN. When they r inclin. at 60°, the " " $5\sqrt{6}$ KN. Then determine the magnitude of the indivisual forces?

A.) 4 KN, 6 KN B.) 6 KN, 8 KN

C.) $\sqrt{50}$ KN, $\sqrt{50}$ KN D.) $\sqrt{25}$ KN, $\sqrt{75}$ KN

Solⁿ: $\theta = 90°$, $F_R = 10$ KN

$\theta = 60°$, $F_R = 5\sqrt{6}$ KN

$$(10)^2 = F_1^2 + F_2^2$$

$$F_1^2 + F_2^2 = 100 \quad \text{—(1)}$$

$$(5\sqrt{6})^2 = F_1^2 + F_2^2 + \overset{\cos 60°}{\left(\tfrac{1}{2}\right)} F_1 F_2 \times 2$$

$$150 = 100 + \tfrac{1}{2} F_1 F_2$$

$$F_1 F_2 = \cancel{100}50 \quad \text{—(2)}$$

$$\therefore F_2 = \frac{50}{F_1}$$

Put F_2

$$F_1^2 + \left(\frac{50}{F_1}\right)^2 = 100$$

$$F_1^4 + 2500 = 100 F_1^2$$

$$F_1^4 - 100F_1^2 + 2500 = 0$$

$$F_1^2 = \frac{100 \pm \sqrt{(100)^2 - 4(2500)}}{2}$$

{let $F_1^2 = x$} $\therefore x^2 - 100x + (50)^2$ (OR) $(x-50)^2 = 0$

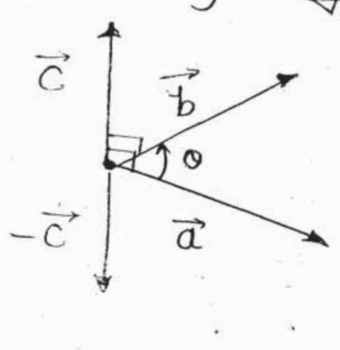
$$F_1^2 = \frac{100}{2} = 50$$

$$\therefore \boxed{F_1 = \sqrt{50} \text{ KN}} \text{ Ans.}$$

$$\therefore \boxed{F_2 = \frac{50}{\sqrt{50}} = \sqrt{50} \text{ KN}} \text{ Ans}$$

CROSS PRODUCT

The dirⁿ of the cross product is always ⊥ʳ to the plane wch contains vector $\vec{a}$ & $\vec{b}$ & its dirⁿ is given by "Right Hand Thumb Rule (RHTR)".



$$\vec{C} = \vec{a} \times \vec{b} = ab\sin\theta \cdot \hat{n}$$

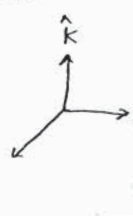
$\hat{n}$ → R.H.T.R.

Unit Vector →

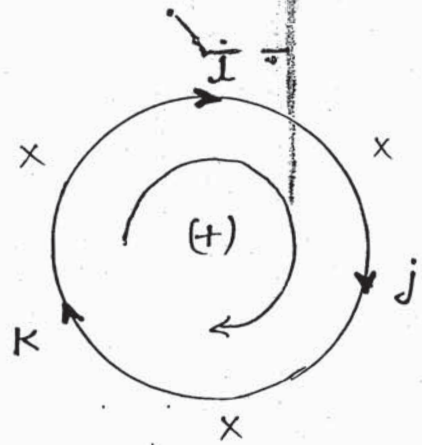
$$\hat{i} \times \hat{i} = 1 \times 1 \times \sin 0 = 0 = \hat{j} \times \hat{j} = \hat{k} \times \hat{k}$$

$$\hat{i} \times \hat{j} = 1 \times 1 \times \sin 90° \, \hat{k} = \hat{k}$$

$$\hat{j} \times \hat{i} = 1 \times 1 \times \sin 90° (-\hat{k}) = -\hat{k}$$



$\hat{j} \times \hat{k} = 1 \times 1 \sin 90^\circ \cdot \hat{i} = \hat{i}$



*
$$\boxed{\begin{array}{c} \vec{a} \times \vec{b} \neq \vec{b} \times \vec{a} \\ |\vec{a} \times \vec{b}| = |\vec{b} \times \vec{a}| \end{array}}$$

Ex: $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{b} = 3\hat{i} - 4\hat{j} + 2\hat{k}$

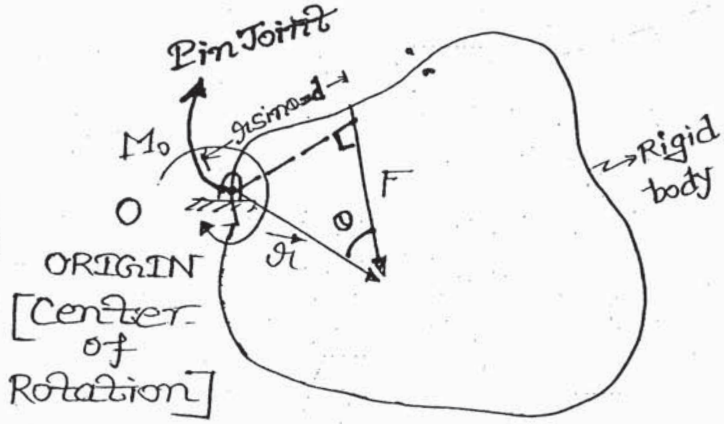
$$\vec{C} = \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & -4 & 2 \end{vmatrix} = i(6+16) + j(12-4) + k(-8-9)$$

$$\vec{C} = 22\hat{i} + 8\hat{j} - 17\hat{k}$$

{i.e. $\vec{C}$ is $\perp^r$ to $\underset{\text{Force Vector}}{\vec{a}}$ & $\underset{\text{Position Vector}}{\vec{b}}$}

TORQUE (or) MOMENT

$$\vec{M}_o = \vec{r} \times \vec{F} = \underbrace{r}F\underbrace{\sin\theta \cdot \hat{n}} = F\underline{d}$$

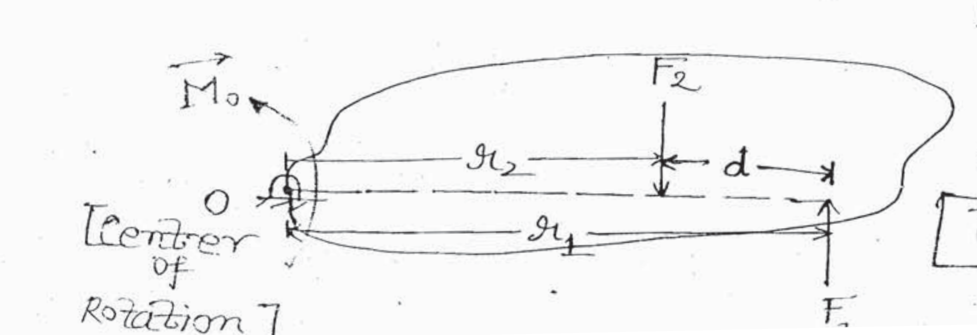


$$\left\{ \begin{array}{l} \quad\quad\underline{\text{For}} \\ \sin\theta = \perp^r \\ \cos\theta = \text{Projection} \end{array} \right\}$$

COUPLE

Couple is an arrangement of 2 equal & opp. forces acting $\parallel^{el}$ to each other & its moment remain uniform throughout the body.

$M_o = r_1 F_1 - r_2 F_2$

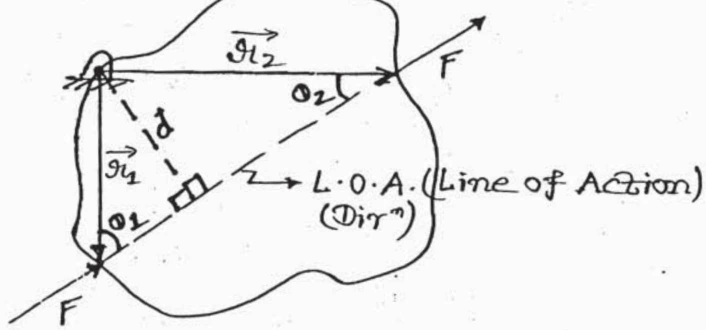


if $F_1 = F_2 = F$

$M_o = \underline{F}(r_1 - r_2)$

$\boxed{C = Fd}$ Anticlock wise Couple

LAW OF TRANSMISSIBILITY



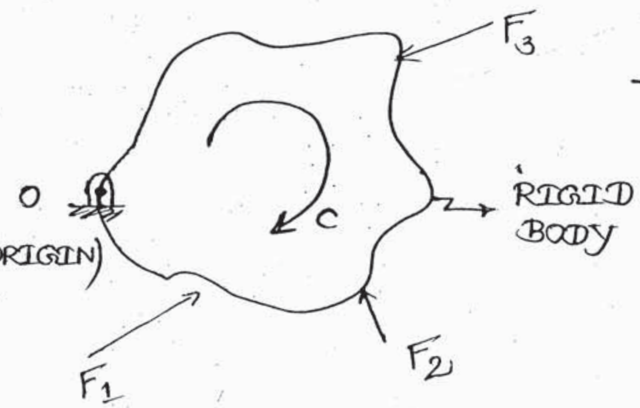
* Truss always at static eqⁿ.

STATIC EQUILIBRIUM

If a body is in static eqⁿ then all d forces & the moment abt any pt. is zero.

- Free Body Diagram [F.B.D.] →

It is the Diagram of an Isolated body by showing all the external as well as internal forces & support rxⁿ by removing support.

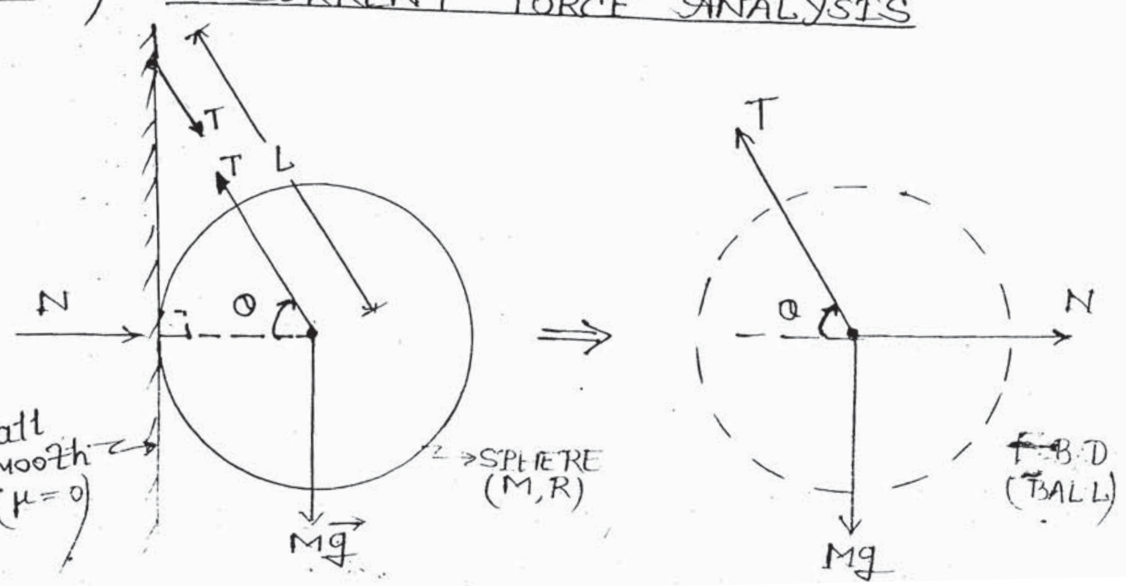


Static - Equilibrium

$$\therefore \Sigma\vec{F} = 0 \Rightarrow \begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \\ \Sigma F_z = 0 \end{cases}$$

$$\Sigma\vec{M_0} = 0$$

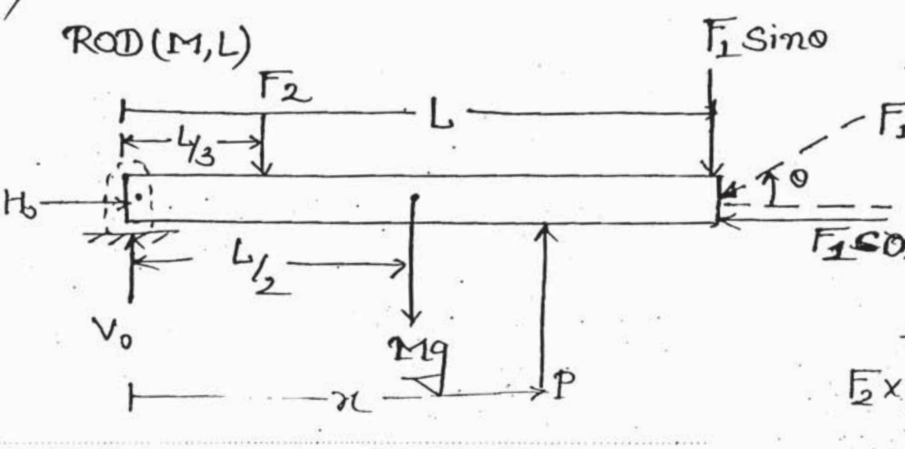
Ex. 1) 'CONCURRENT' FORCE ANALYSIS



$\underline{\Sigma F_x = 0}$ $\qquad$ $\underline{\Sigma F_y = 0}$

$T\cos\theta = N \quad -(1)$ $\qquad$ $T\sin\theta = Mg \quad -(2)$ $\qquad$ $\left\{ \cos\theta = \frac{R}{L} \right\}$ $\underrightarrow{T}$ i.e. find

2) 'NON-CONCURRENT' FORCE ANALYSIS



$\underline{\Sigma F_x = 0}$

$H_0 = F_1\cos\theta$

$\underline{\Sigma F_y = 0}$

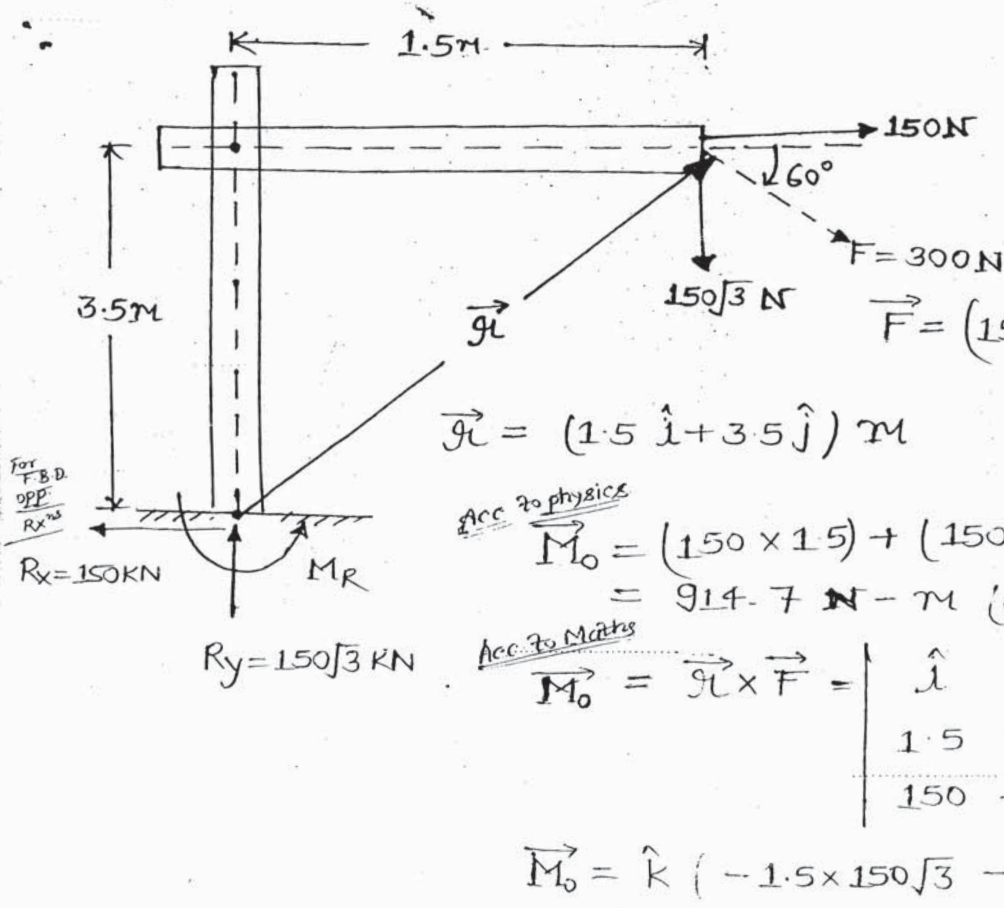
$F_2 + F_1\sin\theta + Mg = V_0 + P$

$\underline{\Sigma M_0 = 0}$

$F_2 \times \frac{L}{3} + Mg \times \frac{L}{2} + F_1\sin\theta \times L = P \times x$

IAS (C.E.)
(2M)

Q.1 Determine the moment abt. a pt. 'O' due to a force acting at the end of a railway sign board post as shown in fig. below –



$\vec{F} = (150\hat{i} - 150\sqrt{3}\hat{j})$ N

$\vec{r} = (1.5\hat{i} + 3.5\hat{j})$ m

Acc. to physics

$\vec{M_0} = (150 \times 1.5) + (150\sqrt{3} \times 3.5)$

$= 914.7$ N-m (C.W)

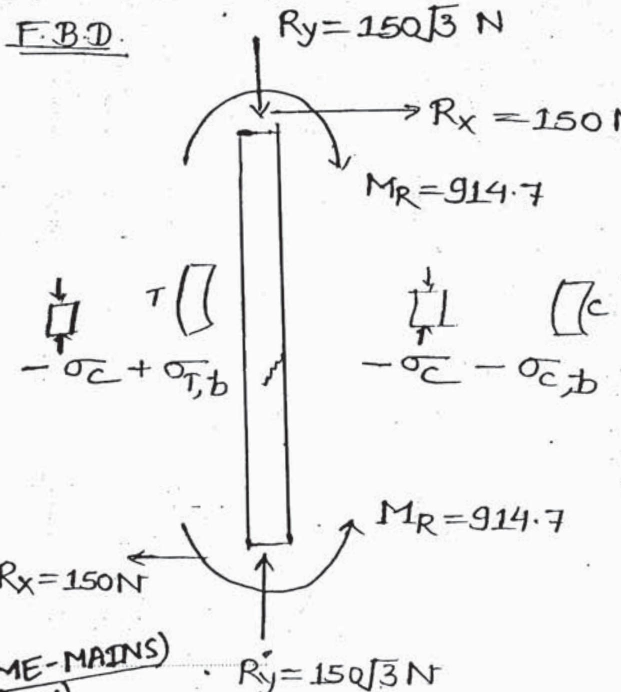
Acc. to Maths

$$\vec{M_0} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1.5 & 3.5 & 0 \\ 150 & -150\sqrt{3} & 0 \end{vmatrix}$$

$\vec{M_0} = \hat{k}\,(-1.5 \times 150\sqrt{3} - 150 \times 3.5)$

$\vec{M_0} = 914.7\,(-\hat{k})$ N-m $\rightarrow$ (C.W.)

F.B.D.

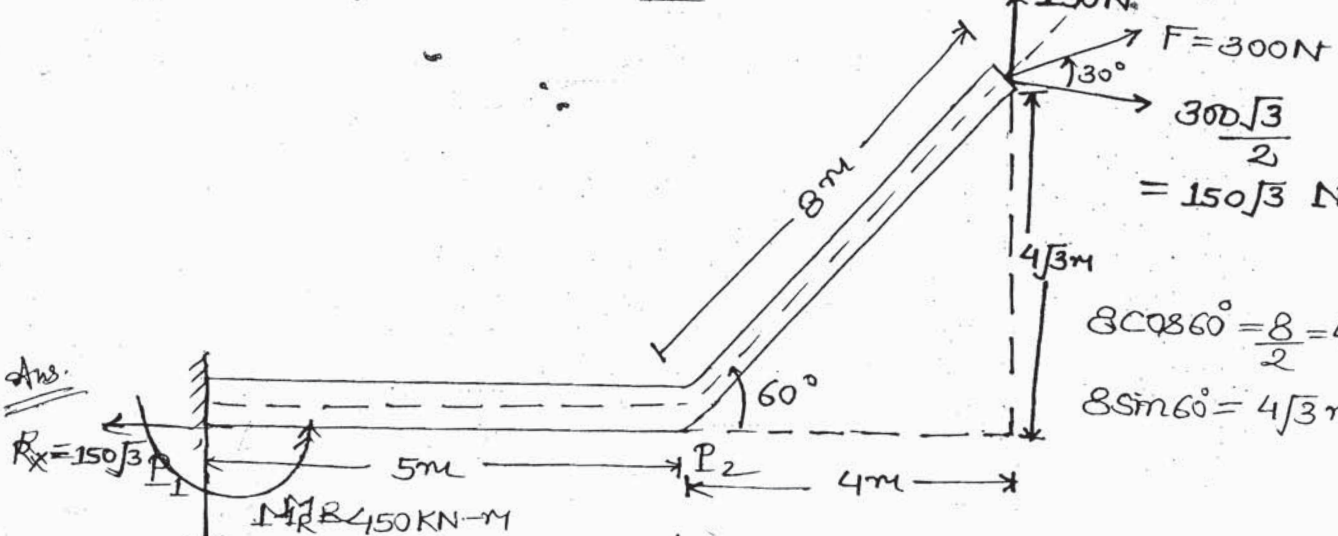


$\sigma_{T,b}$ = Tension due to bending

$\sigma_{c,b}$ = Compressive —〃—

(ME-MAINS)
(12M)
ESE 2018

Q.2 A Rigid [Bracket] body has been loaded as shown in the fig. below determine the moment abt. d Pt. P_1 & P_2. Also find d support rx^n at d fix joint



$8\cos 60° = \frac{8}{2} = 4m$

$8\sin 60° = 4\sqrt{3}\,m$

$\vec{F} = (150\sqrt{3}\,i + 150\,j)$

$\vec{M}_{P_2} = (150\sqrt{3} \times 4\sqrt{3} - 150 \times 4) = 1200\ N\text{-}m\ (C.W)$

$\vec{M}_{P_1} = (150\sqrt{3} \times 4\sqrt{3} - 150 \times 9) = 450\ N\text{-}m\ (C.W)$

$\vec{r}_{P_1} = \overset{(5+4)}{9}\,\hat{i} + 4\sqrt{3}\,\hat{j}$

$\vec{r}_{P_2} = 4\,\hat{i} + 4\sqrt{3}\,\hat{j}$

$\vec{F} = (150\sqrt{3}\,\hat{i} + 150\,\hat{j})$

ESE-18
(CE-MAINS)
(12M) Q.