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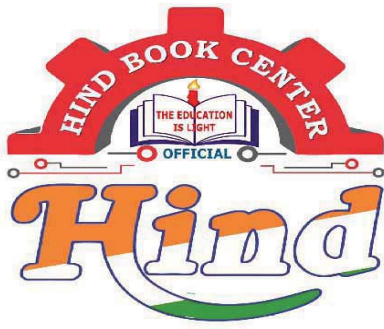
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ENGINEERING MATHEMATICS.

— Dinesh Sir.

Tropic Singh

LINEAR ALGEBRA [MATRICES]

• Properties of Determinant

- 1). If 2 rows/columns of a matrix are identical, then their determinant is zero.

$$\Delta = \begin{vmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & 1 & 2 \end{vmatrix} = 0$$

- 2). If 2 rows/columns of a matrix are interchanged, the ~~value~~^{sign} of determinant is changed.

$$\Delta = \begin{vmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{vmatrix} \quad \text{then} \quad \begin{vmatrix} 3 & 4 & 5 \\ 0 & 1 & 2 \\ 6 & 7 & 8 \end{vmatrix} = -\Delta$$

- 3). If 3 rows/columns of a matrix are interchanged, then the sign of determinant is unaltered.

$$\Delta = \begin{vmatrix} 3 & 4 & 5 \\ 6 & 7 & 8 \\ 0 & 1 & 2 \end{vmatrix}$$

- 4). In the determinant of a matrix, if any column containing the sum or difference of 2 elements, then it can be split into sum or difference of two determinants.

$$\begin{vmatrix} a & a^2 & a^3+1 \\ b & b^2 & b^3+1 \\ c & c^2 & c^3+1 \end{vmatrix} = \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} + \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}$$

- 5). Determinant of :

$$\boxed{|KA| = k^n |A|}$$

where $k \Rightarrow$ scalar

$A \Rightarrow$ matrix of order $n \times n$

$$6). \quad A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \implies \Delta = ad - bc$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \implies \Delta = a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

7). Lower Triangular Matrix: If all elements above the principal diagonal are 0, then it is said to be L.T.M.

$$\text{L.T.M } \Delta = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{bmatrix}$$

8). Upper Triangular Matrix: If all elements below the principal diagonal are 0, then it is said to be U.T.M.

$$\text{U.T.M.} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

Note: If a Matrix is either L.T.M or U.T.M, then determinant is the product of principal diagonal elements.

$$\Delta(\text{L.T.M}) = 1 * 3 * 6 = 18$$

$$\Delta(\text{U.T.M}) = 1 * 4 * 6 = 24$$

Ques: Find the determinant of the matrix.

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = \begin{vmatrix} 0 & a-b & a^2-b^2 \\ 0 & b-c & b^2-c^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c) \begin{vmatrix} 0 & 1 & a+b \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$$

minimum
(n-1) operatⁿ
can be
performed
only.

$$= (a-b)(b-c) (b+c - a - b)$$

$$= (a-b)(b-c)(c-a)$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$$

Note: $|A| = |A^T|$

Ques. Find the determinant of :

$$\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} = \begin{vmatrix} 1 & a & a+b+c \\ 1 & b & a+b+c \\ 1 & c & a+b+c \end{vmatrix} = (a+b+c) \begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix} = \underline{0}$$

$C_3 \rightarrow C_3 + C_2$

Ques. Find the determinant of :

$$\begin{vmatrix} \frac{1}{a} & a & bc \\ \frac{1}{b} & b & ca \\ \frac{1}{c} & c & ab \end{vmatrix} = \frac{1}{abc} \begin{vmatrix} bc & a & bc \\ ca & b & ca \\ ab & c & ab \end{vmatrix} = 0$$

Ques. Find determinant of :

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+a & 1 \\ 1 & 1 & 1+b \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & a & 0 \\ 0 & 0 & b \end{vmatrix} = \underline{ab}$$

$R_2 - R_1, R_3 - R_1$

$$\therefore \begin{vmatrix} 1 & 1 & 1 \\ 1 & 5 & 1 \\ 1 & 1 & 6 \end{vmatrix} = 4 \times 5 = \underline{20}$$

Ques. Find the determinant of :

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc \begin{vmatrix} 1+\frac{1}{a} & \frac{1}{a} & \frac{1}{a} \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & 1+\frac{1}{c} \end{vmatrix} \quad \text{--- } abc$$

$$= abc \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) \begin{vmatrix} 1 & 1 & 1 \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & 1+\frac{1}{c} \end{vmatrix}$$

$$= abc \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) \begin{vmatrix} 1 & 1 & 1 \\ \frac{1}{b} & 1 & 0 \\ \frac{1}{c} & 0 & 1 \end{vmatrix}$$

$$= abc \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) \dots$$

$$\therefore \begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1+b & 1 & 1 \\ 1 & 1 & 1+c & 1 \\ 1 & 1 & 1 & 1+d \end{vmatrix} = abcd \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}\right) \dots$$

also.
$$\begin{vmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{vmatrix} = 5.$$

Ques. Find the determinant of:

$$\begin{vmatrix} a & a^2 & a^3+1 \\ b & b^2 & b^3+1 \\ c & c^2 & c^3+1 \end{vmatrix} = \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} + \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}$$

$$= abc \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} + 1 \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$= (abc+1)(a-b)(b-c)(c-a).$$

Ques If a, b, c are all different and non-zero.

If $\begin{vmatrix} a & a^2 & a^3+1 \\ b & b^2 & b^3+1 \\ c & c^2 & c^3+1 \end{vmatrix} = 0$, then $abc = ?$

here $(abc+1) = 0$

$abc = -1$

Ques. Determinant of the matrix is:

$$\begin{vmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{vmatrix} \Rightarrow \begin{vmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -5 & -8 \end{vmatrix}$$

$R_3 - 3R_2$

$\Rightarrow -1[-8+10]$

$\Rightarrow -2$

[Also by formula. it get the same ans.]

Trick.

0	1	2	0	1
1	2	3	1	2
3	1	1	3	1

☹

$$\Rightarrow 0 + 9 + 2 - 12 - 0 - 1$$

$$\Rightarrow -2.$$

Jitna bli slow karo 9 min lagega.

Maggi Taiyaar.

Ques: Find the determinant of.

(a) $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

$$\begin{array}{ccc} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{array}$$

$$1 + 8 + 8 - 4 - 4 - 4 \Rightarrow \underline{5}$$

(b) $\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

$$\begin{array}{ccc} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{array}$$

$$3 + 0 - 4 - 0 - 0 + 2 \Rightarrow \underline{+1}$$

(c) $\begin{bmatrix} 1 & 2 & 5 \\ 3 & 1 & 4 \\ 1 & 1 & 2 \end{bmatrix}$

$$\begin{array}{ccc} 1 & 2 & 5 \\ 3 & 1 & 4 \\ 1 & 1 & 2 \end{array}$$

$$7 + 8 + 15 - 8 - 4 - 4 = \underline{4}$$

Note: This formula/Trick is applicable on only 3x3 Matrix.

INVERSE OF A MATRIX:

$$A^{-1} = \frac{\text{Adj } A}{\Delta}$$

Note: \Rightarrow Adj A of 2x2 matrix can be find like.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Ques: Find inverse of the Matrix.

(a) $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

(b) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$$A^{-1} = \frac{1}{4-6} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

$$= -\frac{1}{2} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

(c) $B = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$

$$B^{-1} = \frac{1}{6} \begin{bmatrix} 2 & -4 \\ -1 & 5 \end{bmatrix}$$

(d) $A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$

$$A^{-1} = \frac{1}{1} \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$$

(e) $B = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

$$B^{-1} = \frac{1}{ab} \begin{bmatrix} b & 0 \\ 0 & a \end{bmatrix} = \begin{bmatrix} \frac{1}{a} & 0 \\ 0 & \frac{1}{b} \end{bmatrix}$$

⊙ → Adjacent A for higher order matrix :

(a) Minor of an element : The minor of an element in a square matrix is the determinant of the square submatrix in which the row and the column of the particular element lines to be deleted.

for a Square Matrix

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} \Rightarrow \text{Minor of } 1 = \begin{vmatrix} 0 & 2 \\ 1 & 3 \end{vmatrix} = -2$$

(b) Cofactor of an element = $(-1)^{i+j}$ Minor

i.e. $\text{cofactor} = (-1)^{i+j} \text{Minor}$

∴ Cofactor of 1 = $(-1)^{3+2} (-2) = 2$.

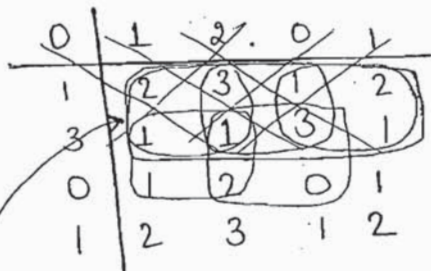
(c) Adj A → Transpose of co-factor matrix.

TRICK :

Ques. Find inverse of matrix of 3x3.

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

$$\Delta = 9 + 2 - 12 - 1 = -2$$



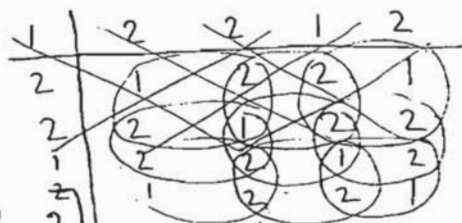
$$A^{-1} = \frac{1}{-2} \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$$

Expand horizontally. don't forget to write vertically.

Ques. Find inverse of :

(a) $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

$$\Delta = 1 + 8 + 8 - 4 - 4 - 4 = 5$$



$$A^{-1} = \frac{1}{5} \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$$