

Linear Algebra:

Determinant value of a square matrix:

The sum of the products of the elements of a row (column) with their corresponding cofactors is known as determinant value of square matrix.

$$2 \times 2 \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc.$$

$$3 \times 3 \begin{vmatrix} 1 & 0 & 1 \\ 2 & -1 & 5 \\ -1 & 2 & 3 \end{vmatrix} = +(-3-10) - 3(6+5) + 1(4-1) \\ = -13 - 33 + 3 \\ = -43.$$

$$\stackrel{(i+j)}{(-1)} \begin{matrix} 4 \times 4 \\ \rightarrow \end{matrix} \begin{vmatrix} 1 & 0 & 2 & 3 \\ 0 & 3 & 0 & 4 \\ 1 & -1 & 2 & 3 \\ 0 & 0 & 0 & 4 \end{vmatrix} = 4 \begin{vmatrix} 1 & 0 & 2 \\ 0 & 3 & 0 \\ 1 & -1 & 2 \end{vmatrix} = 4 \left\{ 1(6-0) + 2(0-3) \right\} \\ = 4(6-6) \\ = 0$$

$$\underline{\underline{4 \times 4}} \begin{vmatrix} 1+x & 2 & 3 & 4 \\ 1 & 2+x & 3 & 4 \\ 1 & 2 & 3+x & 4 \\ 1 & 2 & 3 & 4+x \end{vmatrix}$$

$$C_1 + C_2 + C_3 + C_4$$

$$\begin{vmatrix} 10+x & 2 & 3 & 4 \\ 10+x & 2+x & 3 & 4 \\ 10+x & 2 & 3+x & 4 \\ 10+x & 2 & 3 & 4+x \end{vmatrix}$$

$$R_2 - R_1, R_3 - R_1, R_4 - R_1$$

$\begin{pmatrix} x \\ 0 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 10x \\ 0 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 2 \\ x \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 3 \\ 0 \\ x \\ 0 \end{pmatrix}$	$\begin{pmatrix} 4 \\ 0 \\ 0 \\ x \end{pmatrix}$
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$$= (10+x) \begin{vmatrix} x & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & x \end{vmatrix}$$

$$(10+x)x^3$$

A square matrix A is said to be

- i) Symmetric matrix if $A^T = A$ i.e., $a_{ij} = a_{ji} \forall i, j$
 - ii) Skew symmetric matrix if $A^T = -A$ i.e., $a_{ij} = -a_{ji} \forall i, j$
 - iii) Orthogonal matrix if $A^T A = A A^T = I$

* Every square matrix can be expressed as the sum of symmetric and skew-symmetric matrices.

$$\text{e.g., } A = \left(\frac{A + A^T}{2} \right) + \left(\frac{A - A^T}{2} \right)$$

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Symmetric Skew Symmetric

Let

$$A = \begin{bmatrix} 2 & 5 \\ 9 & 7 \end{bmatrix} \quad A^T = \begin{bmatrix} 2 & 9 \\ 5 & 7 \end{bmatrix}$$

$$\frac{A - A^T}{2} =$$

$$\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$$

skew symmetric

$$a_{ij} = -a_{ji}$$

Always leading
diagonal elements
will be
zero...

Properties:

- * $|A^T| = |A|$

- * $|AB| = |A||B|$

- * $|A+B| \neq |A| + |B|$

- * The det. value of a triangular or a diagonal matrix is the product of its leading diagonal elements..

$$A = \begin{bmatrix} 2 & 3 & 5 \\ 0 & 4 & 6 \\ 0 & 0 & 8 \end{bmatrix}$$

$$2 \times (32-0) + 3 \times 0 + 5 \times 0 \\ = 64$$

Triangular
matrix

$$|A| = 64 \\ = 2 \times 4 \times 8 \\ = 64.$$

- * In a square matrix if each element of a row (column) is zero then the value of its determinant is zero..

Eg. $A = \begin{bmatrix} 2 & 9 & 8 \\ 6 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$

$$|A| = 2(0-0) - 9(0-0) + 8(0-0) \\ = 0$$

* In a square matrix if two rows (columns) are proportional/ identical then the value of its determinant is zero.

Eg. $\begin{bmatrix} 2 & 3 & 5 \\ 6 & 9 & 8 \\ 6 & 9 & 8 \end{bmatrix}$

$$|A| = 2(72-72) - 3(48-48) + 5(54-54) \\ = 0$$

* The determinant value of skew symmetric matrix of odd order is always zero.

$$A = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 3 \\ -2 & -3 & 0 \end{bmatrix}$$

3×3

$$|A| = 0 - 1(0+6) + 2(-3-0) \\ = 0$$

* The determinant value of an orthogonal matrix is always either 1 or -1.

$$AA^T = I$$

$$|AA^T| = |I|$$

$$|A||A^T| = 1 \longrightarrow \therefore |A|^2 = |A^T|$$

$$|A||A| = 1$$

$$|A|^2 = 1$$

$$\boxed{|A| = \pm 1}$$

* If A is a square matrix of order n and K is any scalar then $|KA| = K^n |A|$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2 \times 2}$$

$$KA = \begin{bmatrix} Ka_{11} & Ka_{12} \\ Ka_{21} & Ka_{22} \end{bmatrix}$$

$$|KA| = \begin{vmatrix} Ka_{11} & Ka_{12} \\ Ka_{21} & Ka_{22} \end{vmatrix}$$

$$= k^2 \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$= k^2 |A|$$

* If A is non-singular matrix of order n then

$$i) A(\text{Adj } A) = |A| I \quad \rightarrow |A| \neq 0$$

$$ii) A^{-1} = \frac{\text{Adj } A}{|A|} \quad \hookrightarrow \text{inverse exists}$$

$$iii) |\text{Adj } A| = |A|^{n-1}$$

$$iv) |\text{Adj}(\text{Adj } A)| = |A|^{(n-1)^2}$$

$$v) |A^{-1}| = \frac{1}{|A|}$$

$$\text{Let } A = \begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix}$$

$$|A| = 15 - 14 \\ = 1$$

$$|A| \neq 0$$

Cofactor
matrix

$$= \begin{bmatrix} 3 & -2 \\ -1 & 5 \end{bmatrix}$$

$(-1)^{1+2} \times 2 = -2$

$$\Rightarrow \text{Adj } A = \begin{bmatrix} 3 & -7 \\ -2 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj } A}{|A|}$$

$$= \frac{1}{1} \begin{bmatrix} 3 & -7 \\ -2 & 5 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 3 & -7 \\ -2 & 5 \end{bmatrix}$$

Let $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$

$$\begin{aligned}|A| &= -1(1-4) + 2(2+4) - 2(-4-2) \\&= 3 + 12 + 12 \\&= 27.\end{aligned}$$

$$\begin{array}{ccc}M & & \\ \swarrow & \searrow & \searrow \\ L & -2 & 1 & -2 & -2 \\ \searrow & \swarrow & \swarrow & \searrow \\ F & 2 & 2 & -1 & 2 \\ M & -1 & -2 & -2 & 1\end{array}$$

$$\text{Adj } A = \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj}}{|A|} = \frac{1}{27} \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$$

Q If $A_{m \times n}$ and $B_{n \times p}$ are multiplied then the total number of multiplicative and additive operations are needed to get matrix AB .

- a) mpn, mpn
- b) $mpn, mp(n-1)$
- c) $mp(n-1), mpn$
- d) $mpn, mpn-1$

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & \dots \end{bmatrix}_{m \times n}$$

$$B = \begin{bmatrix} b_{11} \\ b_{21} \\ \vdots \\ b_{n1} \end{bmatrix}_{n \times p}$$

$$AB = \begin{bmatrix} a_{11} b_{11} + a_{12} b_{21} + \dots + a_{1n} b_{n1} \end{bmatrix}$$

Total elements

\times	$+$
n	$n-1$
<u>mpn</u>	$mp(n-1)$

Total elements

for mp elements
 m times multiplied
 $(n-1)$ times added...