* DEFINITION OF ANALOG CIRCUIT:-

* A ckt which consists of atleast one electronic device as the major components then that ckt will be Electronic circuit
  i) Ampl^r.
  ii) Rectifier.
  iii) Oscillator.

* ckts can be of 3 types
  i) Analog ckt (Input also analog and output also analog)
  ii) Digital ckt (Input Digital + Output also digital)
  iii) Mixed Electronic ckt (A to D Convertor, D to A Converter).

* ANALOG ELECTRONIC CKT:-

* An Electronic ckt which performs processing of Analog signals or a ckt in which Input and output are Analog signals. Such ckt are called Analog Electronic ckt.
  i) Amplifier.
  ii) Rectifier; etc

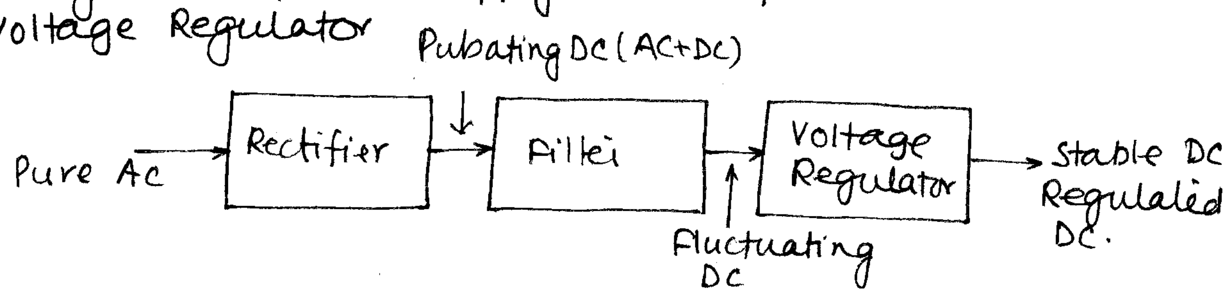
* Real time signals are Analog Signals; hence Analog ckts (usage) ← despite of Digital Era why use Analog ckts.

* Advantages of Analog circuits are:-
  i) Most of the Real time signals are Analog in nature & hence they can be directly processed in Analog circuit. But digital processing requires A to D & D to A Conversion which increases complexity and signal Accuracy is also lost; due to Quantisation Errors.
  ii) Analog ckt can process signals having higher power level also. Digital ckts fails for processing high power supply. Digital ckts often work in mw range.

*Note:- IC's works on DC power supply. They won't work on AC power.

* DC POWER SUPPLY:-

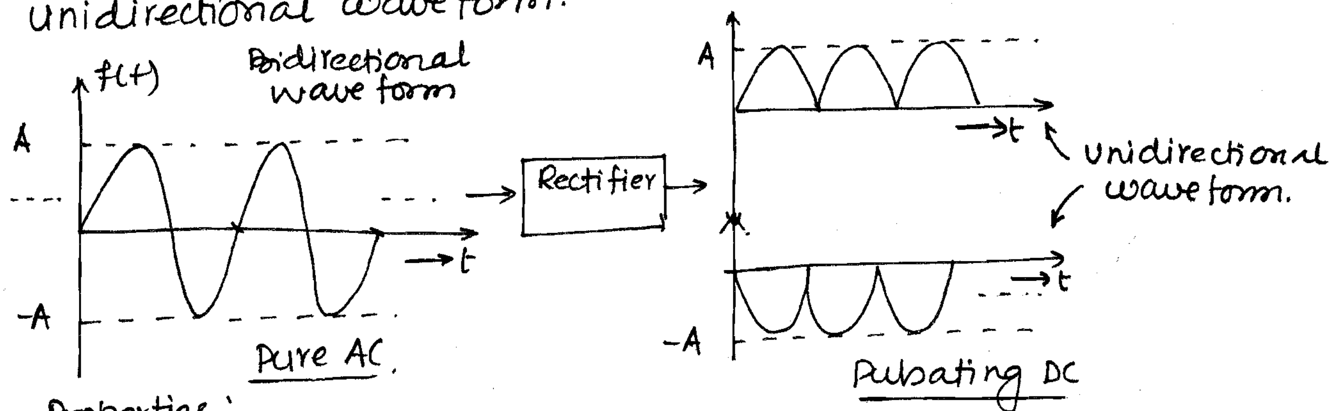
* It converts AC power into DC Power.
* A Regulated power supply consists of a Rectifier, Filter and a Voltage Regulator



* AC to DC conversion is needed because majority of Electronic Devices and appliances operate on DC power.

* RECTIFIER CIRCUIT :-

* An Electronic circuit which converts Pure AC into pulsating DC or a ckt which converts bidirectional waveform into a unidirectional waveform.



Properties :-

i) Periodic variation
ii) Bidirectional variation (both in +ve & -ve values)
iii) Avg. value = 0 (DC value).
iv) It has single frequency component. (Sinusoidal).

* Triangular & Square wave are also called as AC signals but not pure AC as they also have Harmonics. But AC (Pure AC) should have single freqn component.

Properties :-

i) Periodic variation.
ii) Unidirectional variation.
iii) Non Zero Avg, hence DC value will be present.
iv) It has Harmonics.

* Time varying signals have AC components.

Note :-

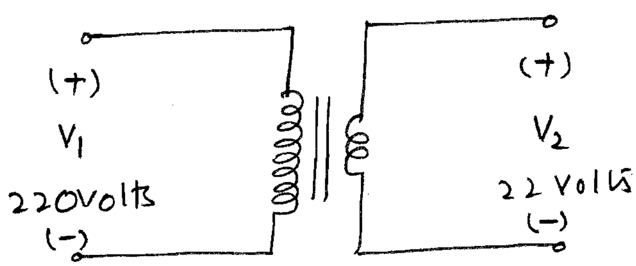
* Periodic variation indicates presence of AC Component that varies with Time
* Non Zero Average indicates presence of DC Component
* Hence Pulsating DC is a combination of AC + DC Components.
* Rectifier Converts Pure AC into Pulsating DC.

*Note:

RECTIFIER

- **Low Voltage Rectifier**
  - DC output Voltage is a few Volt to a few 10's of volts
  - It consists of Diodes
  - Half wave Rectifier
  - Full wave Rectifier
    - Center Tapped Full wave Rectifier
    - Bridge Full wave Rectifier.
- **High Voltage Rectifier**
  - It consists of SCR
  - mainly studied by Electrical Engg in Power Electronics

*Note:
* In low voltage Rectifiers, step down Transformer is used to reduce the strength of AC Voltage



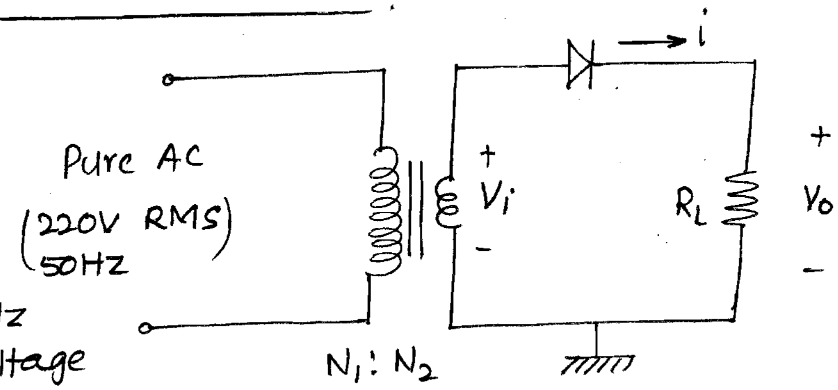
$N_1 : N_2$

10 : 1

$$\boxed{\frac{V_1}{V_2} = \frac{N_1}{N_2}}$$

* Step Down Transformer is needed :
  i) to get low DC Voltage from Rectifier.
  ii) to protect Diodes which have smaller breakdown Voltages.

HALF WAVE RECTIFIER :



20V; 50Hz
→ Line Voltage
→ line frequency

* V_i : Pure AC Voltage having smaller RMS value.

Mathematically,

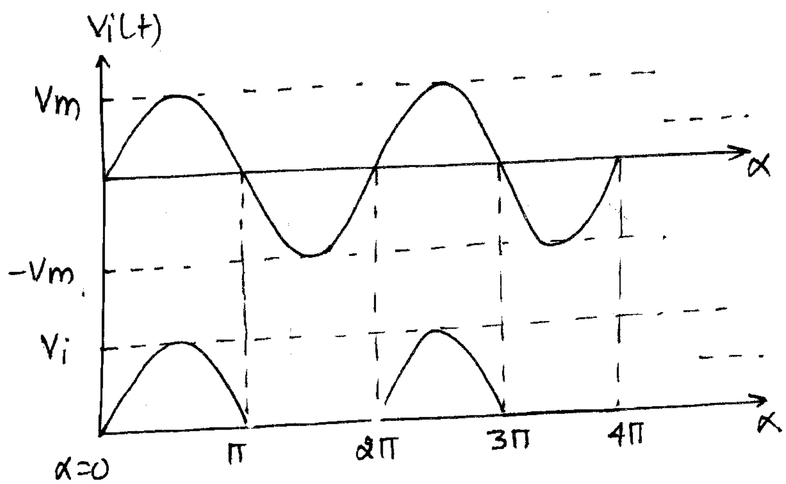
$$\boxed{v_i(t) = V_m \sin\omega_0 t = V_m \sin\alpha}$$

V_m : Peak Value.

$\frac{V_m}{\sqrt{2}}$: RMS value.

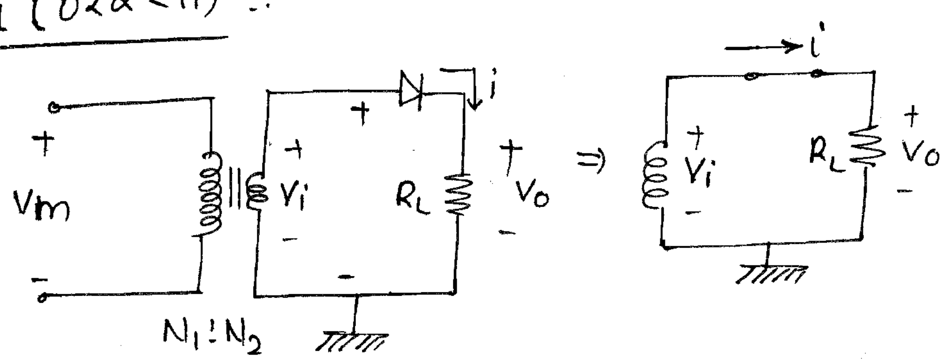
$$\boxed{314\ \text{rad/sec} = \omega_0 = 2\pi f_0}$$

$\boxed{50\text{Hz} = f_0}$: line frequency ie freq^n of AC supply (50Hz).



Analysis :

CASE 1 ($0 < \alpha < \pi$) :

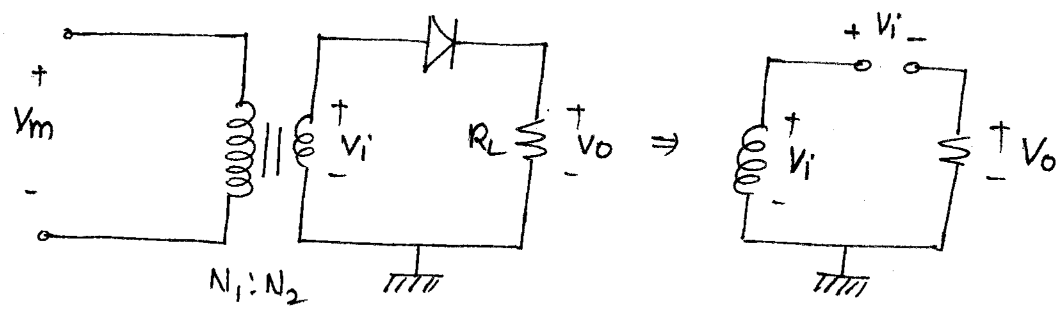


i) V_i is +ve

ii) Diode is in Forward Bias ⇒ Short ckt

iii) $\boxed{V_0 \cong V_i}$

CASE ($\pi < \alpha < 2\pi$):-



* Input Voltage appears fully accross diode which is acting as open ckt

i) Vi become -ve

ii) Diode is in Reverse Biased ⇒ open circuit

iii) $\boxed{V_0 = 0}$

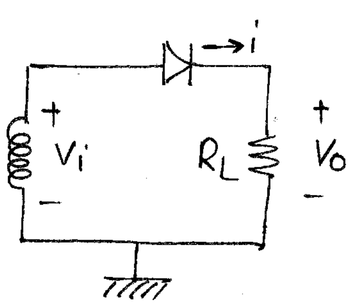
* **Analysis of Half wave Rectifier**:-

i) **Instantaneous output current (i)**:-

a) $0 < \alpha < \pi$ [Diode is in FB ≡ R_F (few Ω)] :-

R_F = Bulk Resistance of Diode (Internal Resistance of Diode).

* R_F : Internal Resistance of Diode; we name technically as Bulk Resistance.



* KVL in Secondary ckt:-

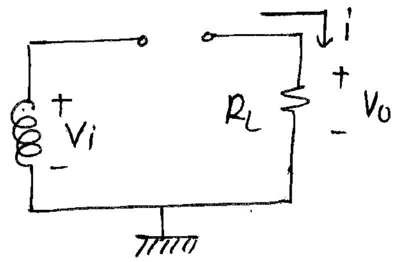
$$-V_i + i \cdot R_F + i R_L = 0$$

$$\boxed{i = \frac{V_i}{R_F + R_L} = \frac{V_m \sin\alpha}{R_F + R_L}}$$

$\boxed{i = I_m \sin\alpha}$; $\boxed{I_m = \frac{V_m}{R_F + R_L}}$

i) $\pi < \alpha < 2\pi$ (Diode is in RB) :.

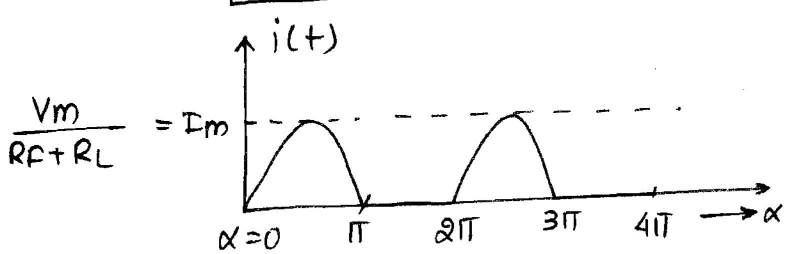
* If a diode is in RB, it passes a negligible current equal to Reverse Saturation current.
* Reverse Saturation current = nA (Si)
  μA (Ge).



$$\boxed{i \cong 0}$$

Hence

$$\boxed{\begin{aligned} i &= I_m \sin\alpha \quad ; \quad 0<\alpha<\pi \\ &\cong 0 \quad ; \quad \pi<\alpha<2\pi \end{aligned}}$$



ATE

ii) DC output current (I_{DC}) :.

I_{DC} = Average value of Instantaneous current "i".

Mathematically

$$I_{DC} = \frac{\text{Area}}{\text{Time Period}}$$

$$I_{DC} = \frac{1}{2\pi}\int_0^{2\pi} i \, d\alpha. = \frac{1}{2\pi}\int_0^{\pi} I_m \sin\alpha \, d\alpha + 0.$$

$$= \frac{I_m}{2\pi}\left[-\cos\alpha\right]_0^{\pi} = \frac{-I_m}{2\pi}\left[-1-1\right]$$

$$I_{DC} = \frac{+I_m}{2\pi}\left[1+1\right]$$

***

$$\boxed{I_{DC} = \frac{I_m}{\pi} \text{ A}}$$

G

iii) RMS output current (I_{RMS}):-

I_{RMS} = RMS value of instantaneous current "i".

Mathematically,

$$I_{RMS} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2 \, d\alpha}.$$

$$I_{RMS} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} I_m^2 \sin^2\alpha \, d\alpha}.$$

$$= \sqrt{\frac{1}{2\pi} \times \frac{I_m^2}{2}\left[\int_0^{\pi} d\alpha - \int_0^{\pi} \cancel{\cos 2\alpha}^{\,0} \, d\alpha\right.}.$$

$$= \sqrt{\frac{I_m^2}{2\pi \times 2}\left[\pi\right]}$$

$$= \sqrt{\frac{I_m^2}{4}}$$

***

$$\boxed{I_{RMS} = \frac{I_m}{2} \, A.}$$

iv) RMS value of AC component (I'_{RMS}):-

*output current of Rectifier is a pulsating DC ie (AC+DC).

$$\boxed{i = \text{AC Component} + \text{DC Component}}$$

$$i = i' + I_{DC}$$

$$\boxed{i' = i - I_{DC}} \leftarrow \text{AC component.}$$

I'_{RMS} = RMS value of i'.

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} (i')^2 \, d\alpha}$$

$$(I'_{RMS})^2 = \frac{1}{2\pi}\int_0^{2\pi} (i - I_{DC})^2 \, d\alpha = \frac{1}{2\pi}\int_0^{2\pi} i^2 \, d\alpha + \frac{1}{2\pi}\int_0^{2\pi} I_{DC}^2 \, d\alpha - \frac{1 \times 2}{2\pi}\int_0^{2\pi} I_{DC} \cdot i \, d\alpha$$

$$(I'_{RMS})^2 = \frac{1}{2\pi}\int_0^{2\pi} i^2 d\alpha + \frac{1}{2\pi}\int_0^{2\pi} I_{DC}^2 d\alpha - \frac{2}{2\pi}\int_0^{2\pi} I_{DC}\, i\, d\alpha$$

$$= \underbrace{\frac{1}{2\pi}\int_0^{2\pi} i^2 d\alpha}_{I_{RMS}^2} + \frac{1}{2\pi} I_{DC}^2 (2\pi) - \underbrace{\frac{2 I_{DC}}{2\pi}\int_0^{2\pi} i \cdot d\alpha}_{I_{DC}}$$

$$(I'_{RMS})^2 = I_{RMS}^2 + I_{DC}^2 - 2 I_{DC}^2$$

$$\boxed{(I'_{RMS}) = \sqrt{I_{RMS}^2 - I_{DC}^2}}$$

← An AC Ammeter Connected in series with R_L will record I'_{RMS}. Therefore I'_{RMS} is also known as Reading of AC Ammeter.

Note :-

* I_{DC} is reading of DC Ammeter.

) RIPPLE FACTOR (r) :-

* The unwanted AC component which is present in the O/P of the Rectifier is known as Ripple.
* Ripple factor is a measure of the amount of AC component

Mathematically,

$$\boxed{r = \frac{\text{RMS value of AC component}}{\text{DC component}}}$$

$$\boxed{r = \frac{I'_{RMS}}{I_{DC}} = \frac{V'_{RMS}}{V_{DC}}}$$

As AC component is unwanted, Ripple Factor should be smaller, and Ideally should be zero.

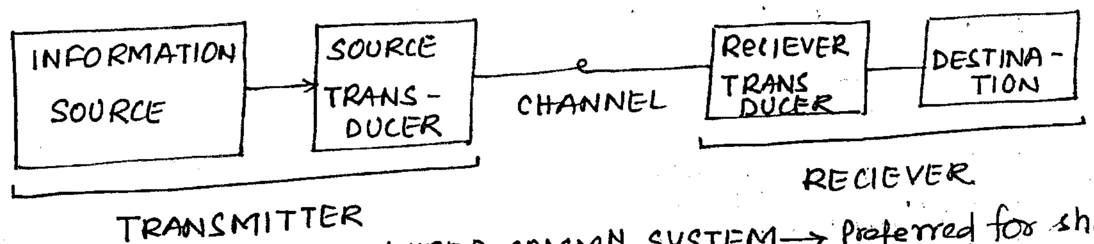
* Analysis :-

$$\boxed{r = \frac{I'_{RMS}}{I_{DC}} = \frac{\sqrt{I_{RMS}^2 - I_{DC}^2}}{I_{DC}} = \sqrt{\left(\frac{I_{RMS}}{I_{DC}}\right)^2 - 1}}$$

* COMMUNICATION:.

* It is the process of transmitting information from source to Reciever.

* BASIC BLOCK DIAGRAM OF COMM^{N} SYSTEM:.

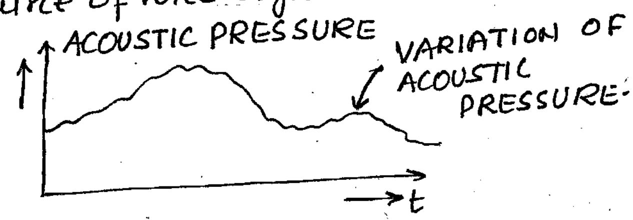


WIRED COMM^{N} SYSTEM → Preferred for short distance

* NOTE:.

i) VOICE SIGNAL:→ Vocal cord is source of Voice Signal.

Range: 300Hz to 3.5 KHz



ii) AUDIO SIGNAL:.

Range: 20Hz to 20KHz.

iii) VIDEO SIGNAL:.

Range: 0 to 4.5MHz

* VOICE SIGNAL is a subset of Audio Signal
* Whatever sound that we can hear is the source of Audio Signal.
* VIDEO SIGNAL → variation of light intensity with time.

Note:.

* Information source is the source of the information.
* Source Transducer converts physical signal into electrical equivalent.

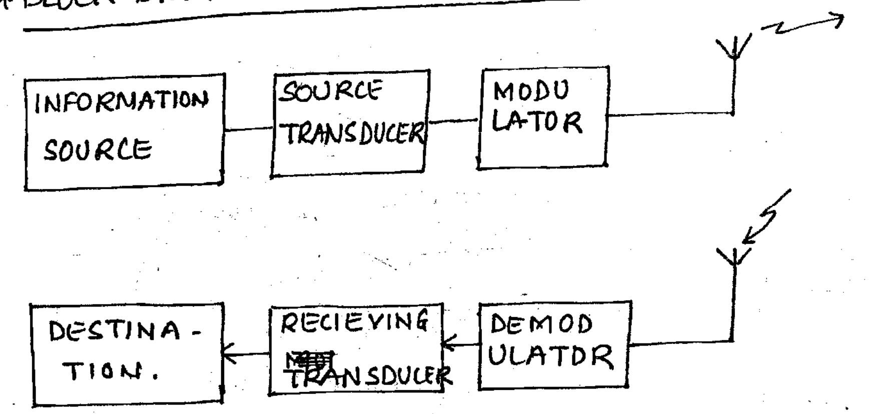
Eg MIC, MICROPHONE.

* Wired communication system is preferred for short distance communication only
* For long distance comm^{n} wireless transmission is preferred in which signal propagates through free space.

*Recieving Transducer converts Electrical signal into Physical equivalent.

Ex: LOUDSPEAKER.

*BLOCK DIAGRAM OF WIRELESS COMM^N SYSTEM!.



*Long distance communication cannot be done without modulation.

* Generally without modulation, long distance communication through free space is not possible

*NEED FOR MODULATION!.

i) Reducing Antenna Height

*For Faithful Radiation the height of Antenna should be

$\boxed{h_t = \lambda/4}$; $\boxed{\lambda = c/f}$ $\Rightarrow$ $\boxed{h_t = \frac{c}{4f}}$

* Faithful Radiation means that the Properties of the Transmitting signal should not change.

Analysis:.

15KHz $\rightarrow$ MOD. $\rightarrow$ 1MHz

let $f_1 = 15\,KHz$

$$h_t = \frac{c}{4f} = \frac{3\times10^8}{4\times15\times10^3}$$

Practically not possible. $\rightarrow$ $\boxed{h_t = 5\,Km}$

$f_2 = 1MHz$

$$h_t = \frac{c}{4f} = \frac{3\times10^8}{4\times10^6}$$

$\boxed{h_t = 75\text{ metre}}$

$\uparrow$

Practically can be implemented.

Note:-

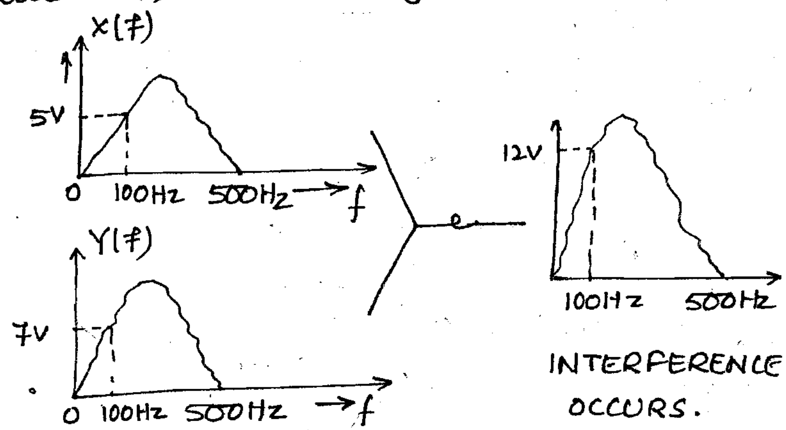
* For Faithful Radiation of a Signal, Antenna Height should be atleast of $\lambda/4$.
* Transmitting Antenna converts ELECTRICAL SIGNAL into ELECTRO MAGNETIC and resulting signal propagates with light velocity.
* MODULATION is the process of increasing frequency of the signal to reduce Antenna height requirements.

II) MULTIPLEXING:-

* Generally without modulation, multiplexing is not possible.
* MULTIPLEXING is the process of transmitting multiple no. of signals through a common channel.
* Generally without modulation, multiplexing is not possible.

$x(t) \longleftrightarrow X(f)$

$y(t) \longleftrightarrow Y(f)$



Note:-

* Due to interference only the interfered signal will be obtained and the original signal is lost in the process.
* Interference process is IRREVERSIBLE. once it occurs, it can't be Reversed ie individual signal can't be obtained back.
* During interference individual frequency components of the original signals are added.
* Due to interference, Multiplexing is failed.
* To avoid this, ~~multiplexing~~ modulation of original signal is done with different carrier frequencies, so that when multiplexed original signal is not lost.

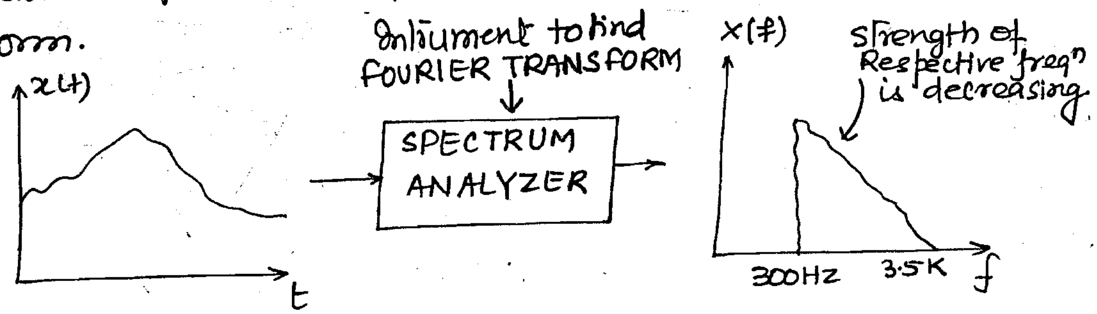
* FOURIER TRANSFORM:

* used to convert time domain signal $x(t)$ to frequency domain signal $X(f)$

$$\boxed{x(t) \longleftrightarrow X(f)}$$

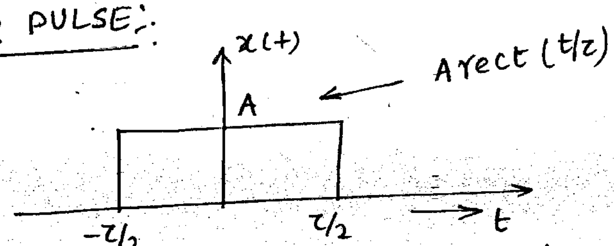
$$\boxed{X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt}$$

* To obtain the frequencies present in $x(t)$ we do its Fourier Transform.



* FOURIER TRANSFORM is basically used to find Frequencies presented in the given TIME DOMAIN SIGNAL.

* RECTANGULAR PULSE:



$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t}\, dt = \int_{-\tau/2}^{\tau/2} A e^{-j2\pi f t}\, dt$$

$$= A \left. \frac{e^{-j2\pi f t}}{-j2\pi f} \right|_{-\tau/2}^{\tau/2}$$

$$= \frac{-A}{j2\pi f} \left\{ e^{-j2\pi f \tau/2} - e^{+j2\pi f \tau/2} \right\}$$

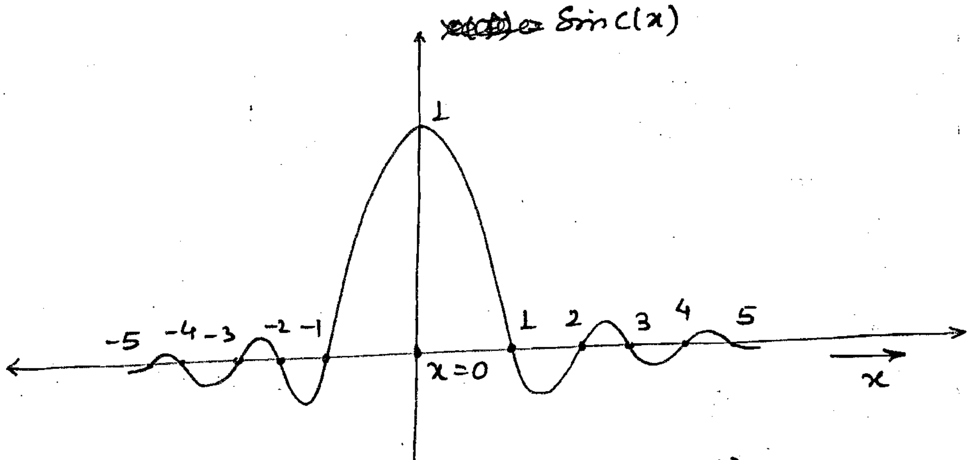
$$= \frac{A}{\pi f} \left\{ \frac{e^{j\pi f \tau} - e^{-j\pi f \tau}}{2j} \right\}$$

$$\boxed{X(f) = \frac{A}{\pi f} \sin(\pi f \tau)}$$

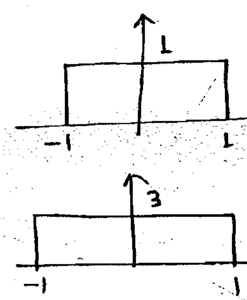
Note :

$$\boxed{\begin{aligned} Sa(x) &= \frac{\sin x}{x} \\ \text{sinc}(x) &= \frac{\sin \pi x}{\pi x} \end{aligned}}$$

$$\boxed{\begin{aligned} \text{sinc}(x) &= 1 \ ; \ x = 0 \\ &= 0 \ ; \ x = \pm 1, \pm 2 \cdots \end{aligned}}$$



Note :



$\leftrightarrow 2\,\text{sinc}(2f)$

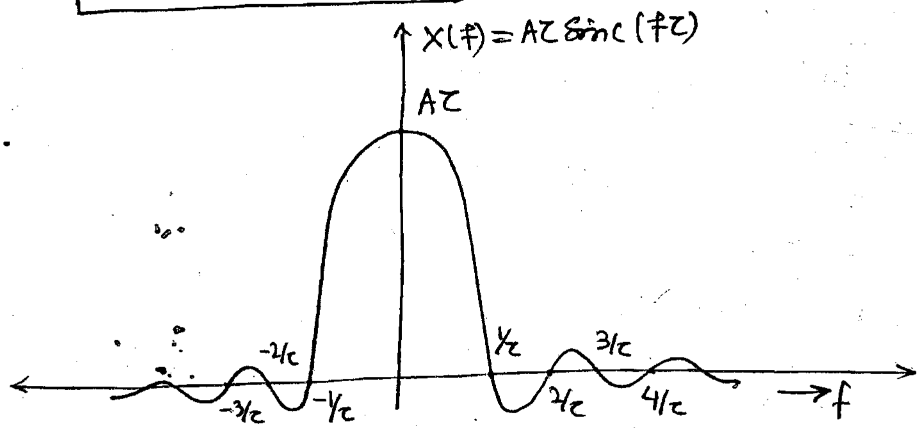
$\leftrightarrow 6\,\text{sinc}(2f)$

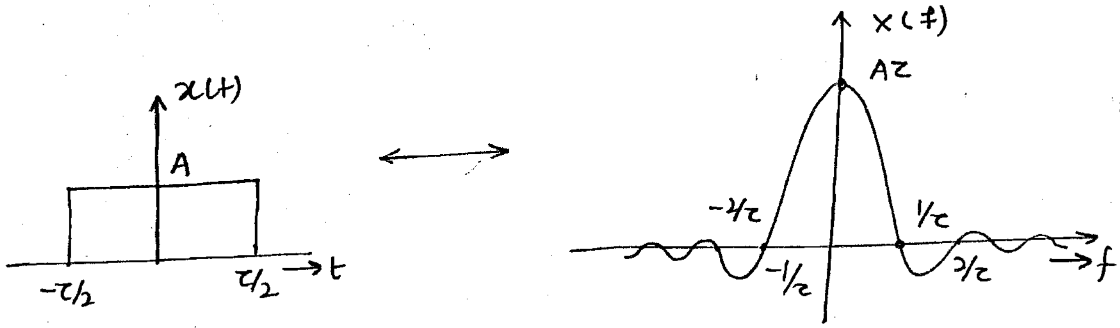
Now, $X(f) = \frac{A}{\pi f} \sin \pi f \tau$

$= A \times \frac{\sin \pi f \tau}{\pi f \tau} \times \tau$

***

$$\boxed{X(f) = A\tau\, \text{sinc}(f\tau)}$$





Note:

* Practically only the +ve frequency exists.
* X(f) contains all possible frequency from 0 to ∞.
* Bandwidth of X(f) is given as:

$$\boxed{BW = \left(\begin{matrix}\text{Highest +ve}\\ \text{Frequency}\end{matrix}\right) - \left(\begin{matrix}\text{Lowest +ve}\\ \text{frequency}\end{matrix}\right)}$$

$\boxed{BW = \infty - 0 = \infty}$ ← of Rectangular Pulse (Ideal Bandwidth).

* Always for faithful transmission:

$\boxed{\text{Bandwidth of channel} \geq \text{Bandwidth of Signal}}$ ← So that Attenuation doesn't occur.

Note (Bandwidth of some Practical channels)

FINITE BAND WIDTH:

i) COAXIAL CABLE ⟶ 0 – 600 MHZ. ← depends on material by which it is made. If material is not good then Bandwidth will be reduced.

ii) PARALLEL WIRE ⟶ 0 – 200 MHZ

iii) OPTICAL FIBRE CABLE ⟶ FEW GHZ

* Bandwidth of channel also depends on its physical dimension.

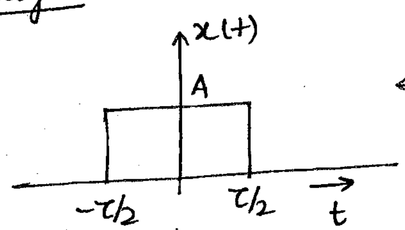
* Every channel (wired) has FINITE BANDWIDTH. Hence the BW of X(f) has to be reduced.

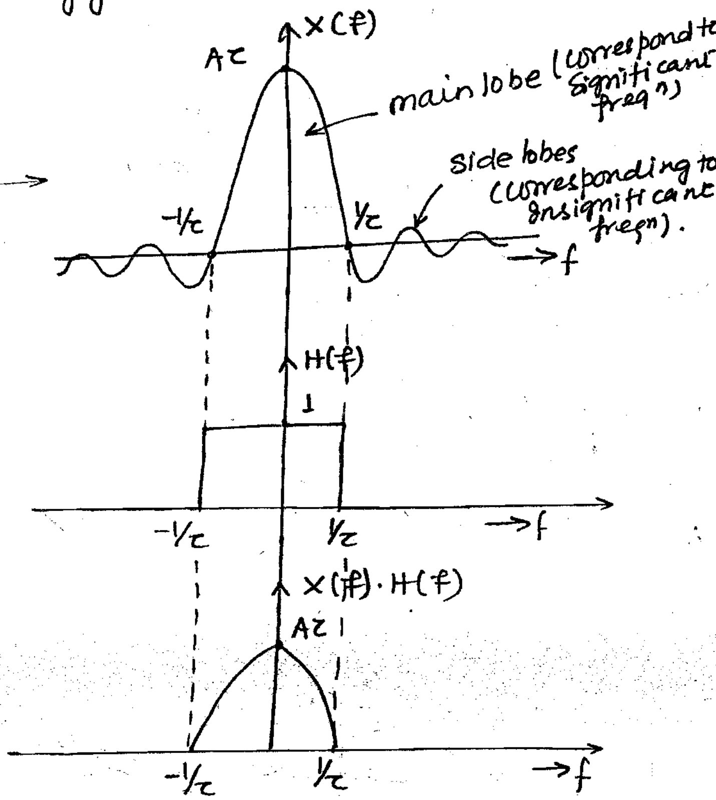
*** BW of FREE SPACE is ∞. Since it is having ∞ BW hence X(f) can be sent to free space but generally not done since in free space there are various frequencies available and then X(f) will get interfered with all those frequencies and will get lost in the process.

Note:

* For Proper Transmission of above signal, channel Bandwidth of ∞ is required.
* But BW offered by Practical channel will be finite only, so that before Transmission above signal should be BANDLIMITED by using "BANDLIMITING PROCESS".
* only those frequency component which contain 95 to 99% of the Energy / Power (total) are kept and rest are discarded during the Bandlimiting Process.
* Significant frequency are those frequencies which contain 95% to 99% of the total energy.

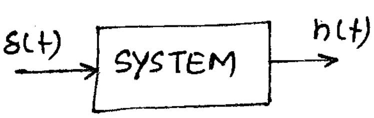
Analysis:





$$\boxed{E = \int_{-\infty}^{\infty} x^2(t)\,dt = A^2\tau}$$

Also, $$\boxed{E = \int_{-\infty}^{\infty} |X(f)|^2\,df}$$

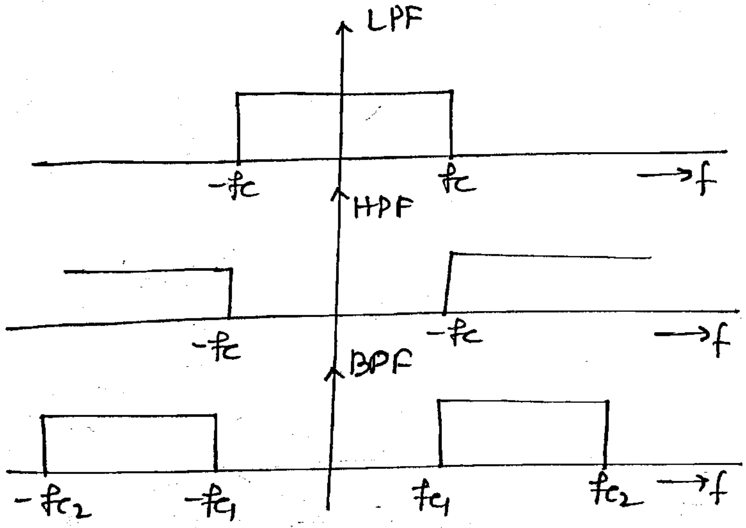


$$\boxed{BW = 1/\tau}$$

$$\boxed{h(t) \longleftrightarrow H(f)}$$

* In Filter Analysis, we take −ve frequency into ~~considered~~ Consideration but in reality they donot exist.

Note :



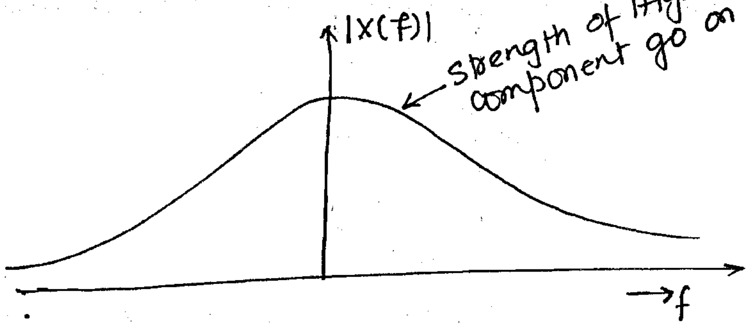
Note :

* In Practical cases only the significant frequencies are to be transmitted. We don't transmit ~~?~~ insignificant frequency
* To Band limit a signal, significant frequencies only ~~???~~ should be retained and insignificant frequencies should be eliminated.
* SIGNIFICANT FREQUENCY CONTAINS 95% to 99% of total strength of signal

*Analysis :

$$x(t) = e^{-t}u(t) \longleftrightarrow X(f) = \frac{1}{a+j2\pi f} = \frac{1}{1+j2\pi f}$$

$$|X(f)| = \frac{1}{\sqrt{1+4\pi^2 f^2}}$$



* Strength of any Naturally generated signal always decreases as frequency increases.
* Naturally occurs ; no mathematical proof.

Transfer Funcⁿ model only for LTI system.

UNIT I:

i) BDR } Graphical
ii) SFG }
iii) Electrical network } Physical
iv) Mechanical system }
v) Integro differential Equation } mathematical
vi) Algebraic Equations }

⇒ Transfer Function.

~~UNIT II~~:-

Analysis of Control system using Transfer function model

- Response Analysis (done for stable system).
  - Time Domain (2)
  - Freqⁿ domain (5)
- Stability Analysis (for any system).
  - Time domain (3) (4)
  - Freqⁿ domain (5).

UNIT 6: (Controller + compensators)

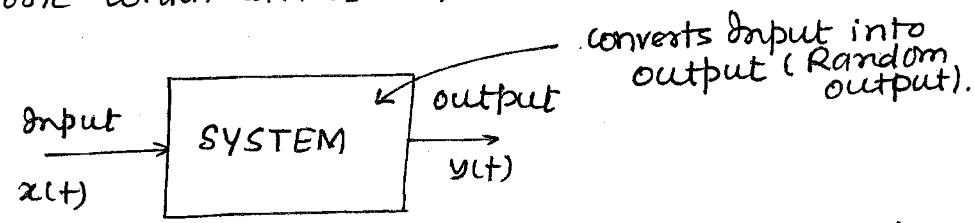
↳ Redesigning of control systems.

UNIT 7:

State model approach. (for any system).

SYSTEM:

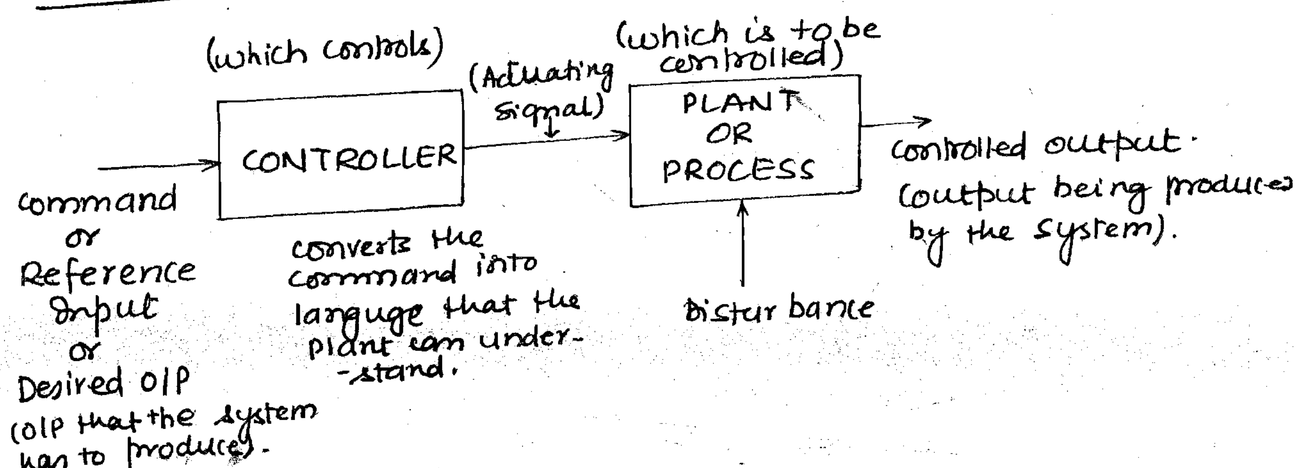
*System is a means of Transforming a signal.
*Signal is one which carries Information.



*control system gives specific output (demanded output). or desired output or deterministic output.

Note:
*control system is that means by which any Quantity of Interest is maintained or altered according to desired manner.

*Block Diagram of control system:

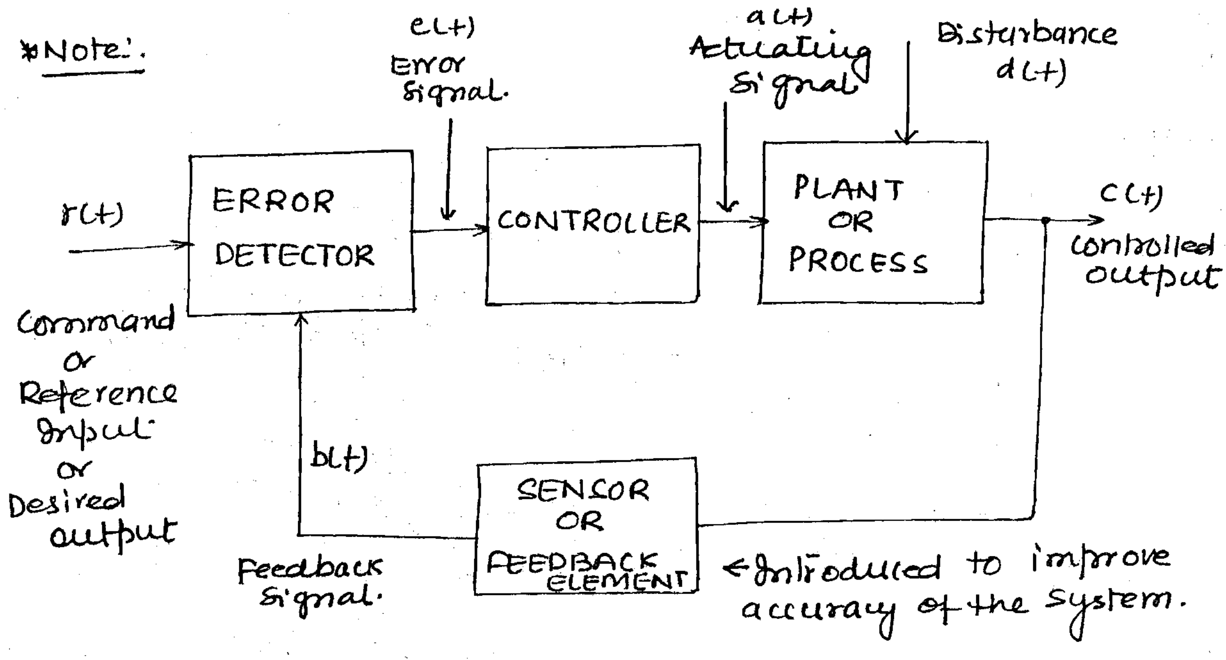


(out to be produced)

IDEAL CONTROL SYSTEM

Note:
*Objective of any control system is to ensure that the controlled output becomes same as the command; or desired output.
*This state of the system is called as STEADY STATE.

***Note:.**



Note:.

* If any disturbance occurs then the output of the control system differs from set value. To Restore the controlled output to its original value; the controll system is modified as shown in above figure.

* Error Detector produces error signal with the help of sensor as the difference between desired output and actual output; which is suppressed by the controller by modifying the output of the Plant. Hence the effect of disturbance associated with the plant disappears from the total output. However, disturbance associated with other parts of the control system still continues in the output of the system which is unavoidable. Hence any practical system can reach the steady state with 100% desired output only at $t=\infty$.

* $\underline{e(t)=0}$; hence rate of change of actuating signal is zero.

$$\boxed{\frac{d}{dt}a(t)=0 \Rightarrow d(t)=K.}$$

Hence output becomes ~~constant~~ constant.

* Feedback in control system is introduced mainly to improve its accuracy but it also has impact on Gain, Bandwidth; speed; sensitivity; stability etc.

* classification of control systems:-

Control systems
- Non Feedback control system (open loop control system) (without sensor)
- Feedback control system (closed loop control system).
  - Human machine (with sensor but without sense. + sense is not enough).
  - Automatic.

* To make OLCS break the feedback connection.

* They can be of 2 types:-

i) with sensor but without sense { used in Real time → Automobile → speedometer doesn't interact with Brakes. ↳ Gap is present since the sense is not enough to drive the process }.

ii) without sensor.

* Differences between performance of open + closed loop control system:-

OPEN LOOP CONTROL SYSTEM	CLOSED LOOP CONTROL SYSTEM.
i) Behaviour of open loop system doesnot change though it's output changes. Hence the open loop system is not accurate.	i) Behaviour of closed loop system does change, if its output changes. Hence closed loop system is accurate.
ii) In open loop system sense is not present/complete, but usually sensor is present not compulsarily.	ii) In closed loop system sence is always present/complete either manually or automatically
iii) Time constant of open loop system is larger due to which the Transients takes large time to die-out. Hence open loop system is slow.	iii) Time constant of closed loop system is smaller due to which transients dies out rapidly. Hence closed loop system is faster.
iv) The effect of external disturbance and internal parameter variation is more in open loop system. ie open loop system is more sensitive.	iv) The effect of external disturbance and internal parameter variations is less in closed loop system ie closed loop system is less sensitive.

v) open loop system is simple + economical.	v) closed loop system is complex and expensive.
vi) open loop system is usually stable but cannot be stabilised if becomes unstable.	vi) closed loop system can become unstable but can be stabalised.

Note:.

* control systems have to stable whether they are:.
  i) linear or non linear
  ii) time variant or invariant
  iii) Static or Dynamic etc.

  } control systems has to be stable whether it may be any of the diff. systems. (L, NL, TV, TI etc).

* stability is necessary in control system since in that condition only we can obtain steady state in which output follows input.

Note:.

* No feedback gaurranteees stability or unstability, -ve F/B always gaurrantees better stability than +ve F/B.
* Inspite of presence of -ve feedback control system can still become unstable due to HIGH OPEN LOOP GAIN; HIGH TYPE NUMBER; HIGH SENSITIVITY; HIGH TRANSPORTATION DELAY OR LAG PHASE.

i) high open loop gain
ii) high type number.
iii) high sensitivity.
iv) High transportation delay or Lag phase.

* Differences b/w the Performance of -ve + +ve feedback closed loop system:

Performance criteria	-ve F/B	+ve F/B
i) Gain → Product const	↓	↑
ii) BW	↑	↓
iii) Time constant	↓	↑
iv) speed.	↑	↓
v) sensitivity	↓	↑
vi) Stability.	↑	↓

Note:.

To analyse the control systems we have a standard models. They are:.

1) Transfer function model.
ii) State model. (latest model 1960).

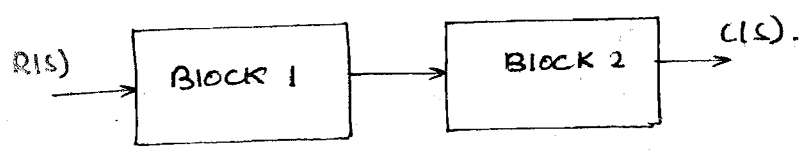
*Transfer function:.
Ratio of Laplace xform of the output and input with initial conditions zero.

*BLOCK DIAGRAM REPRESENTATION:.

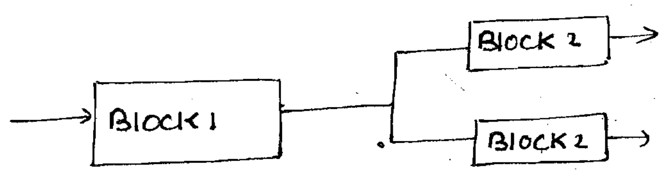
Standard Topologies:.

i) Series/cascade connection
ii) Parallel/Feed forward connection.
iii) closed loop/feedback/canonical connection.

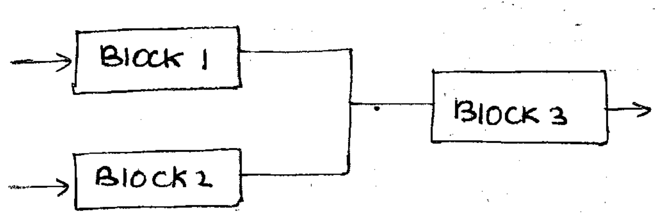
i) series/cascade connection:.



one to one.

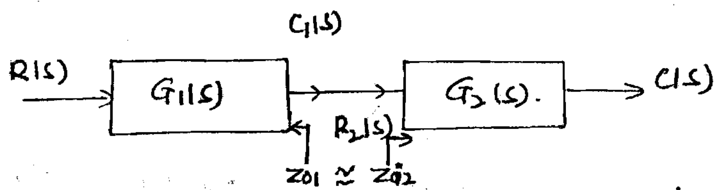


one to many



Many to one.

Note :-



i) $R_2(s) = C_1(s)$ ⟶ Non interactive cascade (no loading effect).

ii) $R_2(s) \neq C_1(s)$ ⟶ interactive cascade (not possible to find TF by BDR but can be found out by electrical network representation

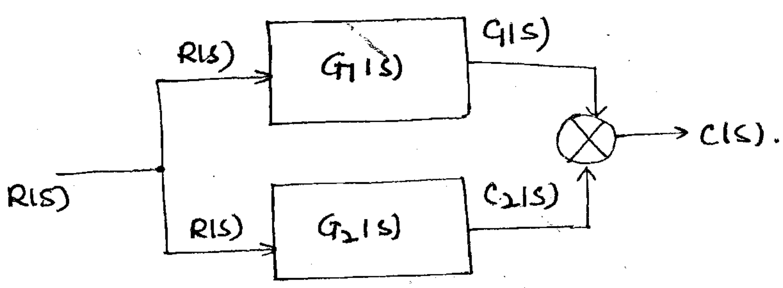
$$C(s) = G_2(s) R_2(s)$$

$$C_1(s) = R(s) G_1(s).$$

$$C(s) = G_2(s) \cdot G_1(s) R(s)$$

$$\boxed{\frac{C(s)}{R(s)} = G_1(s) \cdot G_2(s)}$$

ii) Parallel connection (Topology) :-

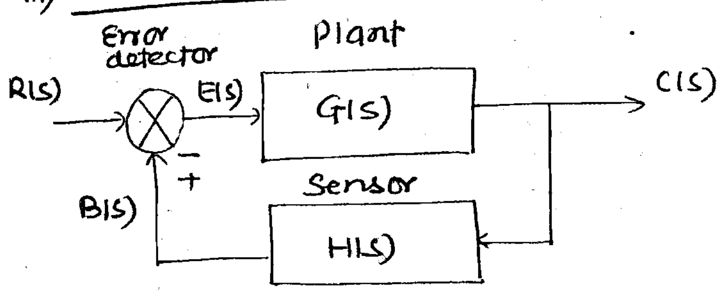


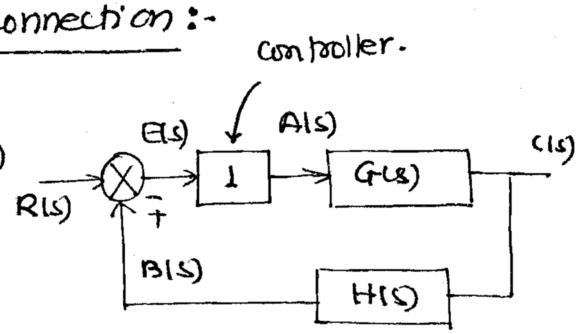
$$C(s) = C_1(s) + C_2(s)$$

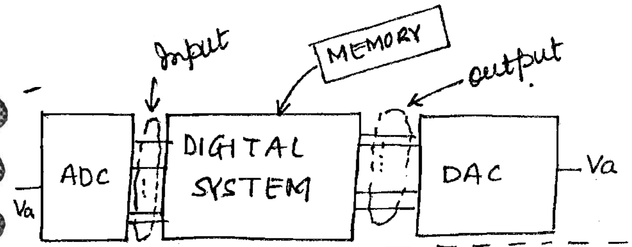
$$= R\,G_1(s) + R\,G_2(s)$$

$$\boxed{\frac{C(s)}{R(s)} = G_1(s) + G_2(s)}$$

iii) Feedback / Closed loop / Canonical connection :-







UP → ALU → Adder & Sub
DSP → Control unit → Decoder or MUX
UC → Registers & Counters → FLIP FLOPS

} LOGIC GATES → BJT OR MOSFET OR DIODES

FPGA → Field Programmable Gate Array.
(Field → The place where we use the device)

SYLLABUS :.

I) Basics
- → Boolean Algebra.
- → Logic Gates.
- → K MAP
- → Number systems, Codes and Data Representation.

II) Combinational circuits :.
i) Arithmetic circuits
- → HA, FA, HS, FS
- → Parallel Adder
- → Look Ahead Carry Adder.
- → BCD Adder.
- → 2's Complement Adder ckt

**ii) MULTIPLEXER (Every Gate & IES paper)...
iii) DEMUX
iv) DECODER
v) ENCODER
vi) COMPARATOR
vii) CODE CONVERTOR
viii) PARITY GENERATORS/CHECKERS

III) Sequential circuits :.
i) Flip Flops.
ii) Registers.
iii) Counters.
iv) State Machines
- → Mealy
- → Moore

Newly Added in GATE

IV) ADC's & DAC's.

V) LOGIC FAMILIES :.
i) RTL
ii) DCTL
iii) IIL
iv) DTL
v) HTL
vi) TTL
vii) ECL

} BJT Based ← Not included in GATE.

i) NMOS
ii) PMOS
iii) CMOS

} FET Based. (Mainly GATE).

VI) Basics of Semi Conductor Memories.
i) RAM.
ii) ROM.
iii) PAL, PLA
iv) PROM.

* Preperation Strategy :-

i) class Notes.

ii) Practising Previous Papers.

- ECE, EE, IN, CSE } GATE
- ECE, EEE } IES.

iii) Reference Books :-

- M. Mano
- Roth.
- Jaub + schilling (ADC & DAC, logic families).

*BOOLEAN ALGEBRA:-

* Introduced in 1854 by GEORGE BOOLE.
* No xtor we available that time, hence designed with help of
  i) VENN DIAGRAM
  ii) SWITCHES → OFF (LOGIC 0) / ON (LOGIC 1)

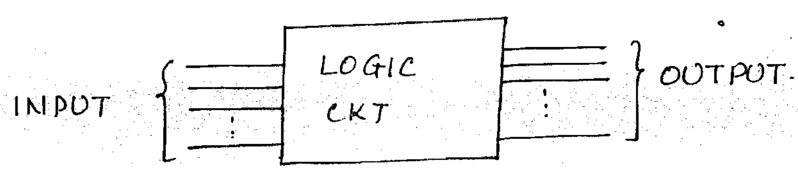
* Boolean Algebra only handles "0 and 1".

A, B, C } VARIABLES → 0 / 1 } Boolean Algebra

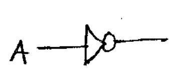
* To minimize logical expressions following methods are used
  i) Boolean Algebra (1,2,3 variables max^m)
  ii) K Map (2,3,4,5 variables at max^m)
  iii) Quine Mc'cluskey or TABULATION METHOD (Any no. of variables)

* Boolean Algebra is used when O/P is either "0 or 1"
  K Map is used when O/P is either "0, 1 or x"

*THEOREMS IN BOOLEAN ALGEBRA:-



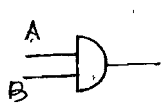
i) NOT:-



$\bar{A} = y$

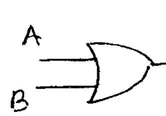
*Note:- $\boxed{\bar{\bar{A}} = A}$ ← NOT operation Relation.

ii) AND:-



$A \cdot B = y$

iii) OR:-



$A + B = Y$

AND OPERATION

$0 \cdot 0 = 0$
$0 \cdot 1 = 0$
$1 \cdot 0 = 0$
$1 \cdot 1 = 0$

OR OPERATION :-

$0 + 0 = 0$
$0 + 1 = 1$
$1 + 0 = 1$
$1 + 1 = 1$

$A \cdot A = A$
$A \cdot 0 = 0$
$A \cdot 1 = A$
$A \cdot \bar{A} = 0$

← AND-OPERATION THEOREM

OR-OPERATION THEOREM

$A + 0 = A$
$A + A = A$
$A + 1 = 1$
$A + \bar{A} = 1$

Q1) Minimize logic expression :-

$$Y = AB + A\bar{B}$$

Soln :-

$$Y = AB + A\bar{B}$$

$$Y = A(B + \bar{B})$$

$$\boxed{Y = A}$$

Q2) To Implement logical exp; $Y = AB + A\bar{B}C + A\bar{B}\bar{C}$; minm no. of 2 Input NAND Gates

a) 0 b) 1 c) 2 d) 3

Soln :-

$$\boxed{Y = AB + A\bar{B}C + A\bar{B}\bar{C}} \leftarrow \text{SOP FORM}$$

$$= AB + A\bar{B}(C + \bar{C})$$

$$= AB + A\bar{B}$$

$$= A(B + \bar{B})$$

$$\boxed{Y = A} \leftarrow \text{NO Gates Required}$$

Q3) Minimize logic expression ; $y = (A+B)(A+C)$

Soln :-

$$\boxed{Y = (A+B)(A+C)} \leftarrow \text{POS FORM.}$$

$$Y = A \cdot A + A \cdot C + A \cdot B + B \cdot C$$

$$Y = A + AC + AB + BC$$

$$Y = A(1 + C + B) + BC$$

$$\boxed{Y = A + BC} \leftarrow \text{SOP FORM}$$

Analysis :.

$(A+B)\cdot(A+C) \rightarrow 3$ variables.

SAME LITERALS

$\boxed{(A+B)(A+C) = A+BC}$

Q4) Minimize ; $(x+y)(x+\bar{y})(\bar{x}+y)$

Soln :. $(x+y\cdot\bar{y})(\bar{x}+y)$

$= x(\bar{x}+y)$

$= xy$

Q5) Minimize ; $(x+y+z)(x+y+\bar{z})$

Soln :. $(x+y+z)(x+y+\bar{z})$

$(x+y)+z\cdot\bar{z}$

$(x+y)$

Q6) Minimize

$Y = (A+B)(A+\bar{B})(\bar{A}+B)(\bar{A}+\bar{B}$

Soln :. $(A+B\cdot\bar{B})(\bar{A}+B\cdot\bar{B})$

$A\cdot\bar{A}$

$= 0.$

Q7) Minimize ; $y = A+\bar{A}B$

Soln :. $y = A+\bar{A}B$

$= (A+\bar{A})(A+B)$

$\boxed{y = (A+B)}$

Note :

$(A+B)(A+C) = A+BC$

$\boxed{(A+BC) = (A+B)(A+C)}$ ← DISTRIBUTION THEOREM.

$(1+2\cdot3) \quad (1+2)\ (1+3).$

Q8) Minimize ;

i) $A+\bar{A}B \rightarrow (A+\bar{A})(A+B) = (A+B)$

ii) $A+\bar{A}\bar{B} \rightarrow (A+\bar{A})(A+\bar{B}) = (A+\bar{B})$

iii) $\bar{A}+AB \rightarrow (\bar{A}+A)(\bar{A}+B) = (\bar{A}+B)$

iv) $\bar{A}+A\bar{B} \rightarrow (\bar{A}+A)(\bar{A}+\bar{B}) = (\bar{A}+\bar{B})$

Q9) Minimum no. of Logic gates required for $y = AB+\bar{A}C+A\bar{B}$

Soln :. $y = AB+\bar{A}C+A\bar{B}$

$= A(B+\bar{B})+\bar{A}C$

$= A+\bar{A}C$

$= (A+\bar{A})(A+C)$

$\boxed{y = A+C}$

Note :- The Precedence order of the LOGIC SYMBOLS is :.

$\boxed{(\ ) > \text{NOT} > \text{AND} > \text{OR}}$ → SOP

$\boxed{(\ ) > \text{NOT} > \text{OR} > \text{AND}}$ ← POS.

Q10) Minimize ; $y = AB + \bar{A}C + BC$

Soln:. $y = AB + \bar{A}C + BC$

Note:. 3 variable Available

↳ Repeated Twice

↳ complement on A & $\bar{A}$

$$\boxed{Y = AB + \bar{A}C + BC = AB + \bar{A}C}$$

$$\boxed{(AB + \bar{A}C + BC) = (AB + \bar{A}C)}$$ ← CONSENSUS THEOREM.

Q11) Minimize logical expression:.

SOP FORM:

i) $\bar{A}B + AC + BC = \bar{A}B + AC$

ii) $A\bar{B} + AC + BC = A\bar{B} + BC$

iii) $AB + AC + B\bar{C} = AC + B\bar{C}$

POS FORM:

iv) $(A+B)(\bar{A}+C)(B+C) = (A+B)(\bar{A}+C)$

v) $(A+B)(A+C)(B+\bar{C}) = (A+C)(B+\bar{C})$

vi) $A\bar{B} + \bar{A}C + \bar{B}C = A\bar{B} + \bar{A}C$

vii) $\bar{A}\bar{B} + \bar{A}\bar{C} + B\bar{C} = \bar{A}\bar{B} + B\bar{C}$

Note:. *check those literals where $A + \bar{A}$ are present ie one Literal A is uncomplemented and $\bar{A}$ is complemented.

* Analysis:.

$$(A+B)(\bar{A}+C) = A\cdot\bar{A} + AC + \bar{A}B + BC$$
$$= AC + \bar{A}B$$

$$\boxed{(A+B)(\bar{A}+C) = AC + \bar{A}B}$$ ← TRANSPOSITION THEOREM.

Q12) Minimize; $y = (A+B)(\bar{A}+\bar{B})$

Soln:. $y = (A+B)(\bar{A}+\bar{B})$

$$\boxed{\begin{aligned} Y &= A\bar{B} + \bar{A}B \\ &= A \oplus B \end{aligned}}$$

Q13) Minimize ; $y = (A+\bar{B})(\bar{A}+B)$

Soln:. $y = (A+\bar{B})(\bar{A}+B)$

$$\boxed{\begin{aligned} Y &= AB + \bar{A}\bar{B} \\ &= A \odot B \end{aligned}}$$

Books :-

1) Semiconductor Physics and Devices
— DONALD NEAMEN.

2) GATE
↳ Basics
↳ Diode
↳ ** FET
} Solved Examples of Donald Neamen.

* CLASSIFICATION OF TEMPERATURE (T) :-

* Divided into three parts :-

1) ABSOLUTE TEMPERATURE (0K = −273°c)
2) ROOM TEMPERATURE (300K = 27°c)
3) AMBIENT TEMPERATURE (T_A) (290K = 17°c)

** $\boxed{°K = K}$ — old Notation: °K, New Notation: K

* Absolute Temperature is Practically not Possible. It is only the Reference Temperature, and never used in Reality.
* Absolute Temperature is just a Reference temperature
* At Room temperature, all properties of semiconductor Devices are max^m at Room temperature.
* All Properties of commn systems are taken at the Ambient Temp. ie 290K or 17°c.

** * $\boxed{\text{TEMPERATURE in KELVIN} = \text{TEMPERATURE in °c} + 273}$

* THERMAL VOLTAGE (V_T) :-

* Also called as the "VOLT EQUIVALENT OF TEMPERATURE".
* Most of S.C devices properties changes with temperature.
* Mathematically,

** $$\boxed{V_T = \frac{\bar{K}\,T}{q}\ \text{volts}}$$

Where, T = Temperature in Kelvin
q = Magnitude of charge (1.6×10^{-19} c)
$\bar{K} = 1.381 \times 10^{-23}$ J/°K

Also, $\boxed{V_T = \frac{T}{11600} \text{ volts}}$

Hence,

i) At $T = 0K \Rightarrow V_T = 0$ volts

ii) At $T = 300K \Rightarrow V_T = \frac{300}{11600}$

** $\boxed{V_T = 0.02568 \text{ volts} = 26 \text{ mV.}}$

Note :

1) For a large variation in Temperature, the variation in the Thermal voltage is negligible.

* **BOLTZMANN CONSTANT** :-

$$\bar{K} = 1.381 \times 10^{-23} J/^{o}K$$

$$K = 8.62 \times 10^{-5} eV/^{o}K$$

Hence, ** $\boxed{\bar{K} = 1.6 \times 10^{-19} K}$

Hence, $V_T = \frac{\bar{K}T}{q} = \frac{q \times KT}{q}$

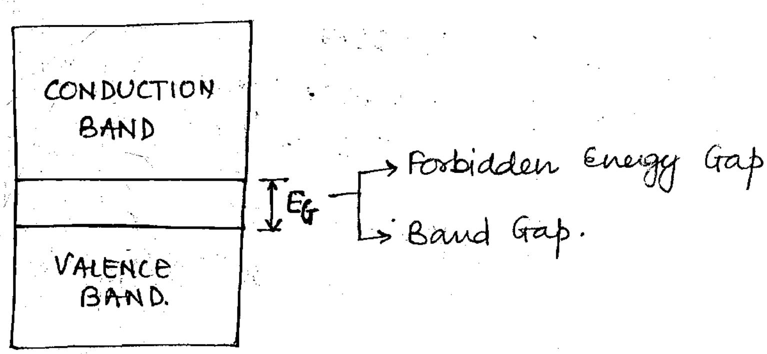
** $\boxed{V_T = KT = \frac{\bar{K}T}{q}}$

↳ Numerically equal values.

* **ENERGY GAP (E_G or E_g)** :-

* Gap between Valence Band and Conduction Band is called as Energy Gap.

* Band diagram of Semiconductor (SC) is given as:-



*

	E_{G0}	E_{G300}
Ge	0.782 eV	0.72 eV
Si	1.21 eV	1.1 eV

**
* Energy Gap decreases with Temperature in a semiconductor.
Mathematically,

**
$$\boxed{E_G \propto \frac{1}{Temp}}$$

* To calculate E_G at different temp we can use:-

**
$$\boxed{E_G(T) = \cancel{E_{G0}}\ E_{G0} - \beta_0 T\ (eV)}$$

β_0 = material constant (eV/°K)

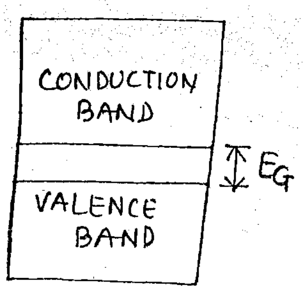
* <u>For Germanium</u>:-

$$\boxed{E_G(T) = 0.782 - 2.33\times10^{-4}\,T\,(eV)}$$

* For Silicon:-

$$\boxed{E_G(T) = 1.21 - 3.6\times10^{-4}\,T\,(eV)}$$

* For a semiconductor, Energy Gap is small
↳ **
$$\boxed{E_G \leq 1.5eV}$$

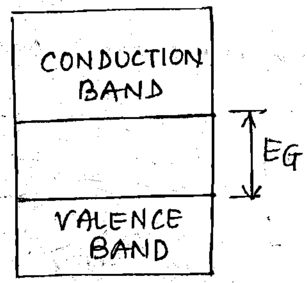


<u>Note</u>:-
1) Semiconductors are BIPOLAR
2) Semiconductor can contribute DIFFUSION CURRENT.
3) Semiconductor has NTC of RESISTANCE

**
$$\boxed{T\uparrow\quad R\downarrow}$$

*Note:

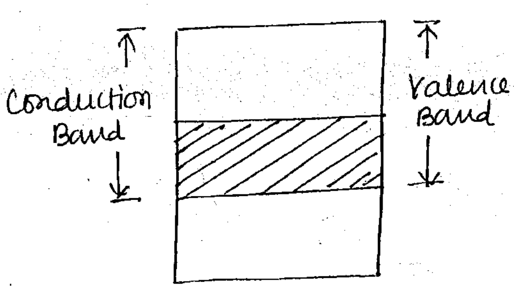
* For Insulators, the Energy Gap is large
  └** $\boxed{E_G \geq 5eV}$



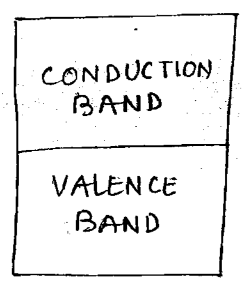
* Insulators are Bad conductors of current, and their Conductivity is negligible.
* For Ideal Insulator, Conductivity is Zero.

Note:
1) If Energy Gap is small, less amount of additional energy is required for the $e^{\ominus}$ to jump from "VALENCE BAND" to "CONDUCTION BAND"

* For Metals (Conductors):
  └ $\boxed{E_G = 0\,eV}$ ← Practically negligible value.
  └ (Non Zero Energy Gap)



(at T= 300K)



(at T=0K)

* For metals, the conductors ~~and~~ the Conduction Band and Valence Band overlap each other and the overlapping increases with Temp.
* Conductivity is very large in Conductors
* Only DRIFT CURRENT flows in Conductor
* Conductors are unipolar, current carried only ~~in~~ by $e^{\ominus}$.
* PTC of Resistance :** $\boxed{T\uparrow\ R\uparrow}$ ← Exclusive Property of Metals.

* Definition of Semiconductor :-

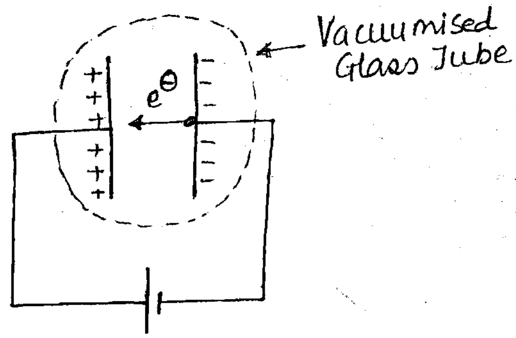
* Semiconductors are the elements whose conductivity lies in between in the conductivity of an insulators and the conductivity of a metal.

* **ELECTRON VOLT (eV):-**

* Electron volt is a unit of ENERGY
* Very small unit of Energy (almost fraction of unit of energy ie Joule).
* Electron volt is the unit of ENERGY in Electronics
** 1 eV is defined as the energy gained by the electron (e^-) in moving through a potential difference of 1V.

Note :-

* Air is a perfect insulator, the Best insulator.



Note :-

* $e^{\ominus}$ cannot move through air, hence air in the glass has been removed.
* $e^{\ominus}$ can move through Vacuum
  → for eg → Vacuum Tubes

Mathematically,

$$1\,eV = |q| \times \text{Potential difference}$$

$$= 1.6\times10^{-19}C \times 1V$$

** $$= 1.6\times10^{-19} C.V$$

Or
$$\boxed{\begin{aligned}1eV &= 1.6\times10^{-19}\text{ Joules}\\ &= 1.6\times10^{-19}\text{ Coulomb-Volt}\end{aligned}}$$

Note :-

* Electron volt is the Kinetic Energy Gained by the $e^{\ominus}$ or the Potential energy lost by the $e^{\ominus}$.

Mathomatically,

** $\boxed{\text{Kinetic Energy} = \frac{1}{2} m v^2 \quad m = \text{mass of } e^{\ominus} = 9.1 \times 10^{-31}\ \text{Kg}}$

** $\boxed{\text{Potential Energy} = q \times V \quad V = \text{Potential difference}}$

By definition:-

KE gained = PE lost

$\frac{1}{2} m v^2 = qV.$

** $\boxed{\text{Velocity of } e^{\ominus},\ v = \sqrt{\frac{2qV}{m}}\ \text{m/s}}$

* ELECTRIC FIELD INTENSITY ($\mathcal{E}$ or E) :-

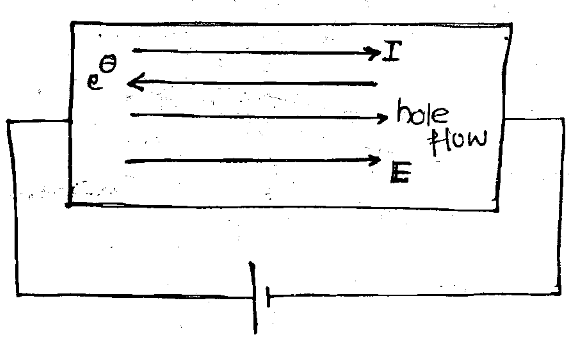
* Also called Field Intensity
* Also called as Field Gradient
* Also called as Field.

* Mathematically,

** $\boxed{\mathcal{E} = -\frac{dV}{dx}\ \text{Volts/metre}}$

Also, ** $\boxed{|\mathcal{E}| = \frac{\text{magnitude of voltage existing}}{\text{distance or space}}}$

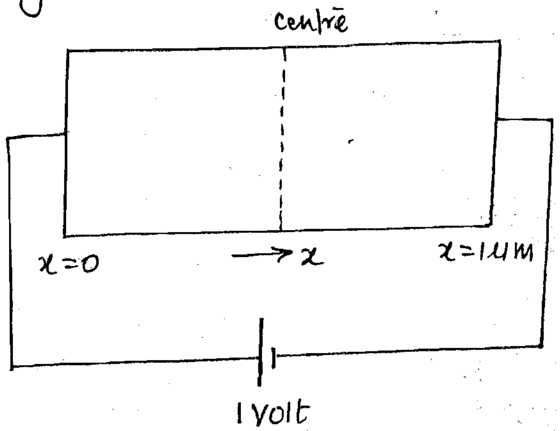
Note :-



* Field is always directed from +ve of the Battery to the -ve of the battery.
* Field is directed from "higher potential to lower potential".

Q1) Considering a uniform semiconductor Bar (Rectangular shape) calculate the field at the centre of the bar?

* specially designed question for Gate (donot change the values).



Soln:. At the centre of the Bar:.

$x = 0.5\mu m$

$V = 0.5\ V.$

Hence

$$|E| = \frac{|V_c|}{x_c}$$

$$E = \frac{0.5V}{0.5\mu m}$$

$$\boxed{E = 1\times10^{6}\ V/m}$$

* MOBILITY OF CHARGE CARRIERS (μ):-

* Mobility of charge carrier is the ability of the charge carrier to move from one place to another ie how fast the charge carrier can move from one place to another.

* MOBILITY is defined:.

** $$\boxed{\mu = \frac{\text{drift velocity}}{\text{field intensity}}}$$

* Also, ** $\boxed{\mu = \frac{v_d}{E}}$ → Unit $m^2/Vsec$ or $cm^2/Vsec$

* MOBILITY denotes how quick is the $e^{\ominus}$ or the hole in moving from one place to another place.

** * DRIFT VELOCITY is the velocity of charge carrier under FIELD INTENSITY

** * DRIFT VELOCITY is the average velocity of charge carriers.

** $$\boxed{v_{drift} = \frac{v_{max} + v_{min}}{2}}$$

* electron mobility = $\mu_e = \mu_n$
  Hole mobility = $\mu_h = \mu_p$

**

	GERMANIUM	SILICON
μ_n	3800 $cm^2/Vsec$	1300 $cm^2/Vsec$
μ_p	1800 $cm^2/Vsec$	500 $cm^2/Vsec$

* $\boxed{\frac{\mu_n}{\mu_p} = 2.1 \text{ (for Ge)}}$ $\boxed{\frac{\mu_n}{\mu_p} = 2.6 \text{ (for Si)}}$

*Note:-

**1) $e^{\ominus}$ mobility (μ_n) is always greater than hole mobility (μ_p) and therefore the $e^{\ominus}$ can travel faster and also contributes more current when compared to the hole.

**2) For higher conductivity and larger currents, Ge devices must be preferred.

Ge
- → Large conductivity (due to larger mobilities)
- → Relatively more suitable for high frequency application (Large Gain Bandwidth Product)

3) Both Ge and Si have smaller ~~leakage~~ switching times ~~currents~~ and Ge has larger leakage currents as compared to Si.

Si
- → Relatively high speed Applications (due to smaller leakage currents)
- → High Power applications.

**4) Switching times are small in Ge and Si. Also,

$\boxed{f = \frac{1}{t}}$; t = switching times

Hence, both Si and Ge can work at high frequency, but Ge is preferred over Si, since Ge has larger "GAIN BANDWIDTH PRODUCT"

SYLLABUS :-

	THEORY	PROBLEMS	PROBLEMS
1) Static Electromagnetic Fields.	(Hayt and Buck);	Sadiku ;	Schaum Series

2) Time varying fields → Electro-Magnetic waves. (JORDAN BALMAIN).
3) Transmission Lines → Voltage and current waves. (JOHN D RYDER).
4) Waveguides (JORDAN BALMAIN).
5) Antennas and Radiated waves. (JORDAN BALMAIN).

Methodology of Preperation:-

VSRS22@gmail.com
facebook ID
VSR suresh.

1) Concepts / Theory / Fundamentals.
2) Application / Questioning style.
3) Beyond classroom
   ↳ Previous Papers → EC (Gate/ESE).
                      → EE (Gate/ESE).

TEXT BOOK :-

1) HAYT & BUCK.
2) SADIKU.
3) JOHN D RYDER.
4) JORDAN BALMAIN.

SESSION 1 :-

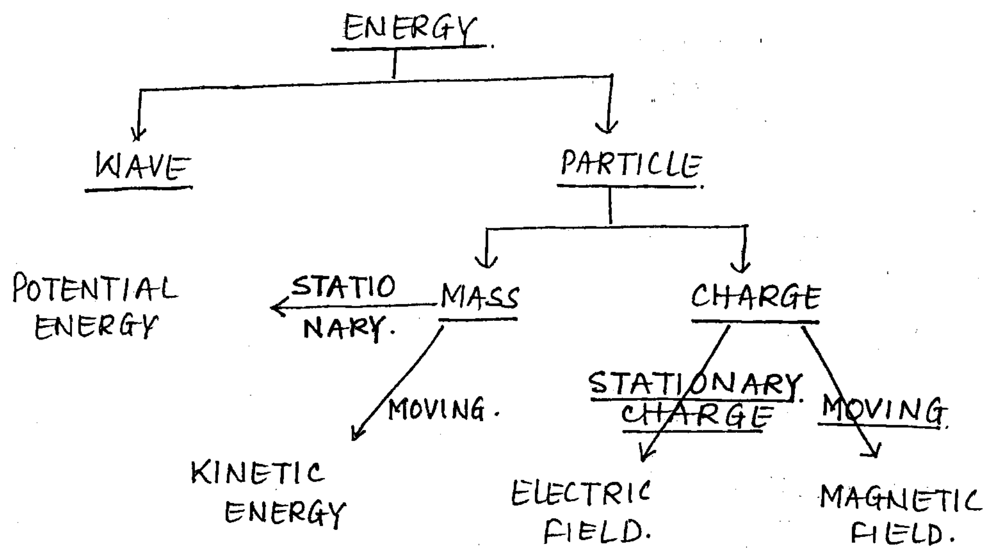
1. Vector calculus.
   * Vector function
   * Density / Intensity funcn
2. Co-ordinate Systems
   * dl, ds, dv
   * (·) Dot
   * (x) Cross.

STATIC ELECTRO-MAGNETIC FIELDS

SESSION 1

*DEFINITION OF FIELD:-

* Everything in this world is ENERGY.



ELECTRIC FIELD:-

*Electric field is a format of Energy that is all around a charge and influences similar charges nearby

Note:- Electric field cannot be seen but can be felt by a test charge when brought in its vicinity.

MAGNETIC FIELD:-

*Magnetic field is a format of Energy that is all around a moving charge and influences similar moving charges nearby

Note:- Magnetic field cannot be seen but can be felt by another moving charge.

Note:-

1) Stationary charge → (CAUSE) VOLTAGE (D.C voltage)

↓

ELECTRIC FIELD (EFFECT)

Note:-

*When voltages are given to the conductors, materials then the effect is seen in the free space.

*Voltages to conductors, materials (2D).

↳ effects the outer space (free space) (3D).

Note:- As in Antennas, where voltages and currents are given to the conductors and they start radiating signals in 3D space

2) Moving charge → DC CURRENT → CAUSE → MAGNETIC FIELD. (EFFECT)

Note:.
* when current is given to the conductor, materials it will give the cause in the free space and that is 3D space.
* Current or Voltage given to Antenna hence felt in free space.

VECTOR CALCULUS:.

* It is the study of DIRECTIONAL INTEGRATIONS and DIRECTIONAL DERIVATIVES in 3 DIMENSIONAL SPACE.

DIRECTIONAL INTEGRATION:.

* It is calculation of the total effect of any phenomenon in a given direction in a given region.
* This can be implemented over a line, over a surface or over a volume. ie

$\int dl$ → line Integral.

$\iint ds$ → surface Integral.

$\iiint dv$ → volume Integral.

DIRECTIONAL DERIVATIVE:.

* Directional derivative is the study of RATE OF CHANGE of any phenomenon in a given direction in a given region.
* Helps in the study of Rate of flow.
* Helps in understanding the nature of variation of any phenomenon.
* DEL OPERATOR is used for study of spatial variations in 3D of space. It is a vector operator.

Mathematically,

$$\boxed{\nabla = \frac{\partial}{\partial x} a_x + \frac{\partial}{\partial y} a_y + \frac{\partial}{\partial z} a_z}$$ ← derivation along with the directions.

** It can be used to study the Rate of change of:

1) Scalar Quantities.
2) Vector Quantities.

* Examples are:

1) $f(x,y,z) = 4x^2y - 5z^3$ ← Scalar Quantity.

2) $\bar{A}(x,y,z)(x,y,z) = 4x^2y\,a_x + 7y\,a_y + 12xz\,a_z$ ← Vector Quantity.

Mag. depends on (x,y,z) — direction depends on (x,y,z)

3) $A(x)(y) = 4x^2 a_y$ ← mag. depends on x.
direction depends on y.

* GRADIENT :

* $\nabla \rightarrow$ operated on scalar function ie ∇f

** $\boxed{\text{Gradient of scalar} \rightarrow \text{Result is vector function}}$

* DIVERGENCE AND CURL :

* ∇ operated on vector function is called as:

1) Divergence → Dot product
2) Curl → Cross Product.

* Divergence of Vector given as $\nabla \cdot \vec{A}$. The Result is a scalar.
* curl of Vector given as $\nabla \times \vec{A}$. The Result is vector.

Mathematically,

** 1) $\nabla \cdot \vec{A}$ = Dot product
= Divergence of Vector
Result of operation is SCALAR.

** 2) $\nabla \times \vec{A}$ = cross product
= curl of Vector
Result of operation is Vector.

Note :-

* $\boxed{\nabla \cdot \nabla = \nabla^2 = \text{Second order derivative}}$ ← called as SCALAR LAPLACIAN operator.

Vector Identity :-

1) $\boxed{\nabla \times \nabla f = \nabla \times (\nabla f) = 0}$

$\boxed{\text{curl of Gradient of Scalar} = 0}$

2) $\boxed{\nabla \cdot (\nabla \times A) = 0}$

$\boxed{\text{Divergence of curl of Vector} = 0}$

Note :-

$A \times B = C$

$C \perp (A \;\&\; B)$

Hence, $A \times C = |A||C| \sin 90^\circ \hat{n}$
$= |A||C| \hat{n}$

$A \cdot C = AC \cos 90^\circ$
$= 0.$

So, $\boxed{A \cdot (A \times B) = 0 \Rightarrow \nabla \cdot (\nabla \times A) = 0}$

Note :-

$A \times B = |A||B| \sin\theta \, \hat{n}$

$A \cdot B = |A||B| \cos\theta$

* $\nabla \times \nabla = 0$; Since both are same vectors and moving in same direction as like $A \times A$. Hence,

$\nabla \times \nabla = |\nabla||\nabla| \sin 0 \, \hat{n} = 0$

So, $(\nabla \times \nabla f) = 0$

* ~~Also, $\nabla \times \nabla \neq 0$; Since both are same vector and moving in same direction.~~

* $\nabla \times A$ results in a vector $\perp^r$ to both ∇ and A. Hence

$\nabla \cdot (\nabla \times A) = \nabla \cdot B$

$B = (\nabla \times A)$ and $B \perp A$
$B \perp \nabla$.

So, $\nabla \cdot B = |\nabla||B| \cos 90^\circ$
$= 0.$

3) $\boxed{\nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A}$

Note :-

1. $\nabla \times (\nabla \cdot A)$
2. $\nabla (\nabla \times A)$

→ not allowed. Since curl of Scalar is not allowed. Also, ~~Divergence~~ Gradient of Vector is not allowed.

3. $\nabla (\nabla \cdot A) = \nabla^2 A$

*OUTFLOW & DIVERGENCE OF VECTOR FUNCTION:.

*Consider a cause or a source, having some effects radially outward from the cause. for all such phenomenon the STRENGTH decreases as the AREA OF EXPANSION increases; such that:.

"The TOTAL OUTFLOW, through any enclosing surface is always a CONSTANT, and this constant depends on the central cause"

*The strength represents a DENSITY VECTOR FUNCTION or closeness of the lines; and mathematically

$$\boxed{\text{strength} = \frac{\text{Constant}}{\text{Area}} = \frac{\text{cause}}{\text{Area}}} \qquad \boxed{\text{Constant} \propto \text{Cause}}$$

*If a cause is of Q coulombs of charge, the effect represents, the Physical attractive or repulsive force on any charge nearby. This is called as Electric flux or electric field.

CAUSE OR SOURCE : Q

EFFECT : Electric Force/Field/Flux (Ψ_e)

STRENTH OF EFFECT : Electric flux Density ($\vec{D}$)

*The strength is ~~cause~~ called as Electric Flux Density ($\vec{D}$) such that:.

$$\boxed{\oiint_{closed} \vec{D}\cdot\vec{ds} = \Psi_e\,(\text{total}) \propto Q.}$$

Note:. The effect around the charge (Q) is called as Electric field and can be felt by test charge and is not visible.

$$\boxed{\oiint_{closed} \vec{D}\cdot\vec{ds} = Q.}$$ ← GAUSS LAW IN INTEGRAL FORM.

Note:.

*If the surface is not completely enclosing, the effects are Partial ie ~~$\oiint D\cdot ds$~~

$$\boxed{\iint_{open} D\cdot ds = \Psi_e \neq Q}$$ ← flux Passing through the surface (open), only through that open surface and this is not GAUSS LAW.

* Every closed Surface is Identified with a finite volume enclosed.

1) $4\pi r^2 \xrightarrow{\text{sphere}} \frac{4}{3}\pi r^3$

2) $2\pi r h \xrightarrow{\text{cylinder}} \pi r^2 h$

3) $6a^2 \xrightarrow{\text{cube}} a^3$

4) $\pi r^2 \xrightarrow{\text{circle}}$ volume not defined.

* Mathematically,

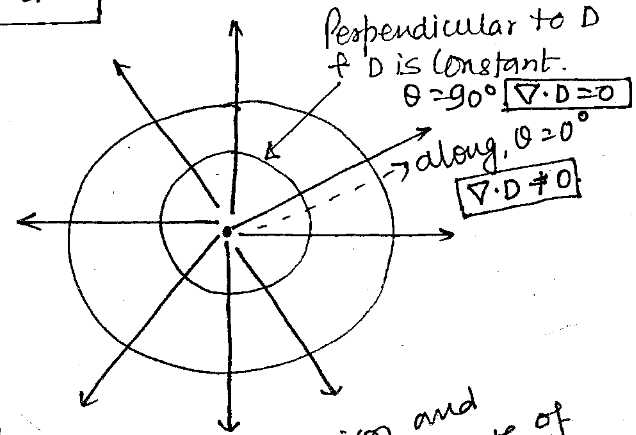
$$\boxed{\text{strength of field, } \vec{D} = \frac{dQ}{ds}} \leftarrow \text{coulombs}/m^2.$$

* $\frac{dQ}{dv} = \frac{d}{dl}\left(\frac{dQ}{ds}\right)$

so, $\frac{dQ}{dv} = \frac{d}{dl}\left(\frac{dQ}{ds}\right) = \nabla \cdot D$

$\rho_v = \nabla \cdot D$

so, $\boxed{\nabla \cdot D = \rho_v}$ ← GAUSS LAW in point form



D's direction and direction of decrease of D (change of D).

Divergence at any point depends on the volume charge density

* The DOT (·) operation in derivative signifies the directional derivative in the vector direction.

Note:

* Rate of change of strength (D) ← ELECTRIC FLUX depends on charge density (ρ_v). ($\nabla \cdot D = \rho_v$)

* $\boxed{\nabla \cdot D = |\nabla||D|\cos\theta}$

* The significance of Dot product is that, to understand the Rate of change of D, we have to read it along D. The surface given above are ⊥r to D. Hence $\theta = 90° \Rightarrow \nabla \cdot D = 0$

Note Cause: Q → Effect = D or E.

* $\nabla \cdot D$ → Represents rate of change of effect
* helps in understanding the cause.
* by finding ρ_v, charge stored in the volume helps in understanding the charge, Q.

MICROPROCESSOR:

i) 8085. (Gate – in detail) → 2M/3M GATE; → 20 Marks to 40 Marks IES.

ii) 8086.

iii) 8051.

Note:

** IAS/IPS
ELECTRICAL OPTIONAL (MAINS)
↓
ECE + EEE

** BEFORE 2011
IAS PRELIMS ELECTRICAL
GATE STANDARD (OBJECTIVE).

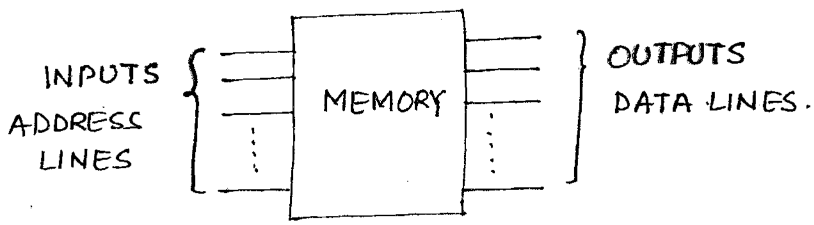
8085 SYLLABUS:

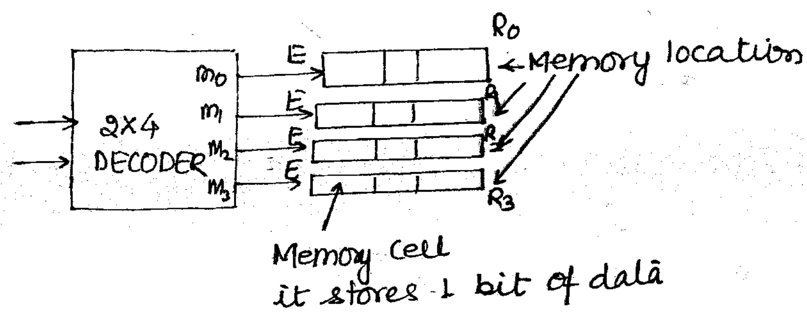
i) Memories.
ii) 8085 Basics.
iii) Instruction set] 1 Marks GATE

iv) Programming
v) Interfacing] ** 2 MARKS GATE

*MEMORIES:

*Used for storage.





Note:

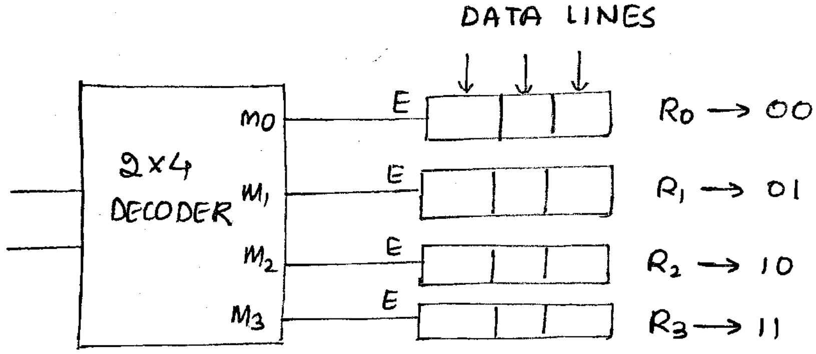
* By giving input 00, m_0 will get selected hence the address of R_0 is 00.

* By giving input 01, m_1 will get selected hence the address of R_1 is 01.

MEMORY LOCATION	ADDRESS
R_0 →	00
R_1 →	01
R_2 →	10
R_3 →	11

* ADDRESS :-

*ADDRESS is a binary code which enables a particular location.



*In order to store data in memory the following sequence has to be followed:-

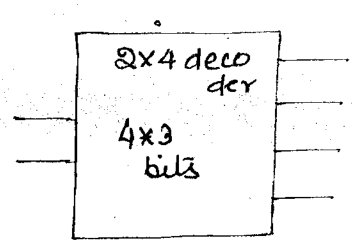
i) Select the location by giving an appropriate address.

ii) give the data through the Data lines.

*SIZE OF

*Size of Memory is measured in bits and is equal to No. of memory location multiplied with No. of bits / location

$$\boxed{\text{Memory Size} = \text{No. of memory location} \times \text{No. of bits/location}}$$

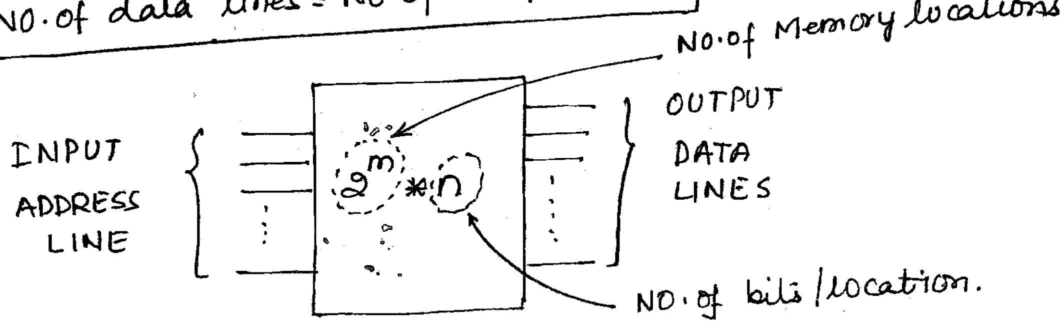


4 → locations.

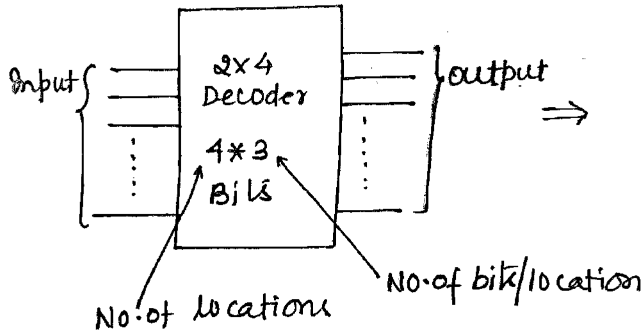
3 → bits / location.

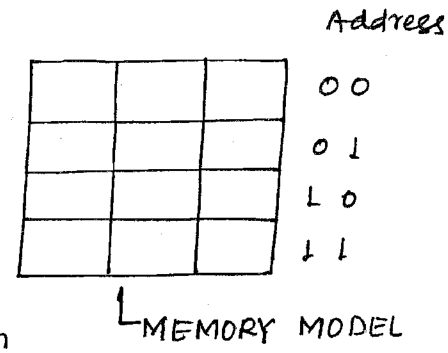
*For m address lines, no. of location is 2^m.

* $\boxed{\text{No. of data lines} = \text{No. of bits / location}}$



* MODEL OF MEMORY :-





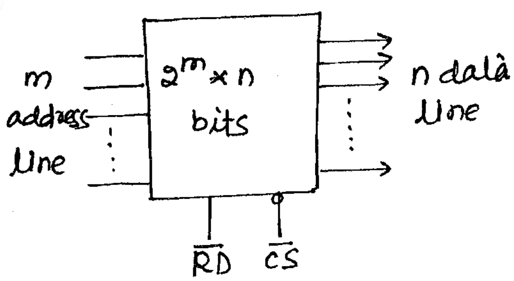
* Two types of Memory :-

i) Read only Memory (ROM)

ii) Read / write Memory (RWM)

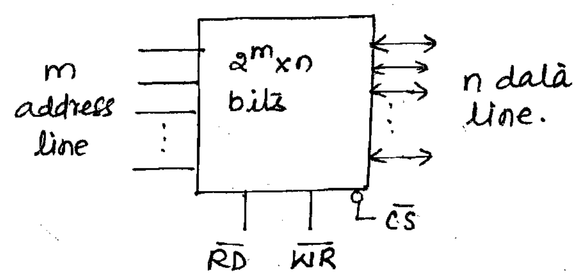
↳ commercially called RAM.

READ ONLY MEMORY :-



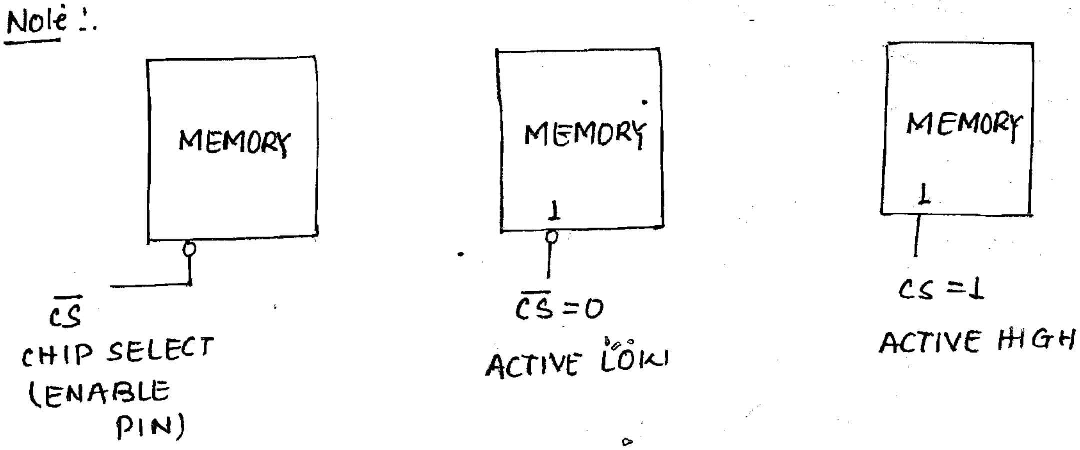
* unidirectional data line to only Read data.

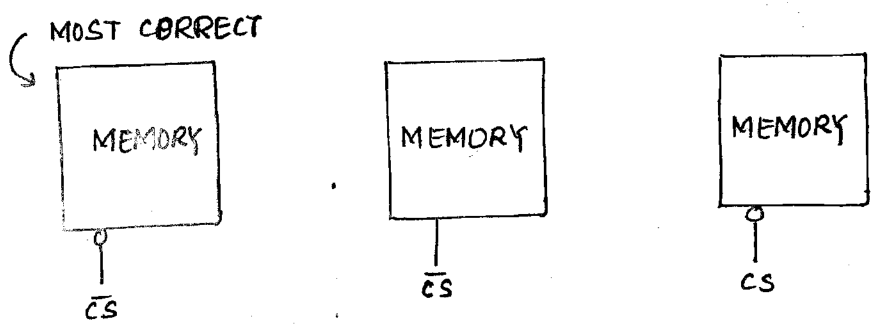
RANDOM ACCESS MEMORY



* Bidirectional data line to Read and write data

Note :-



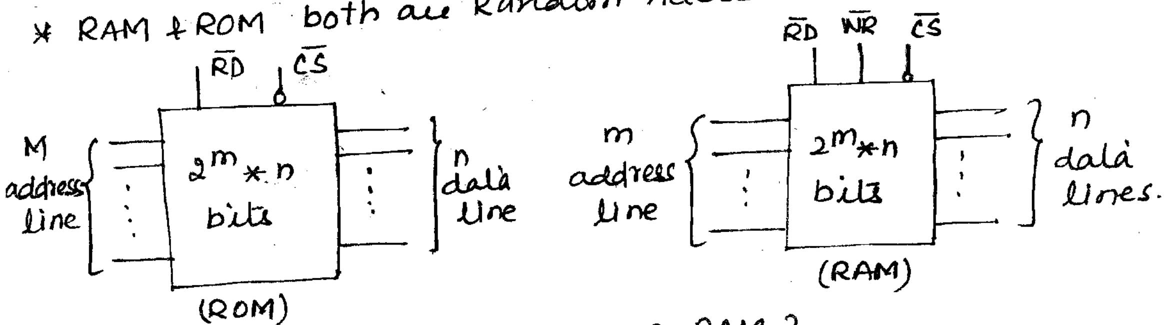


* Note:.
* All are 3 possible ways of Representing CHIP SELECT.

* ~~RAM~~
* Random Access V/s Serial Access:.

* In Random access we directly give the address and reach the location where data is stored, but in Serial access to reach some location we have to go serially

* RAM & ROM both are Random Access.



Q1) construct 8 KB RAM using 2KB RAM ?

Soln:. KILO $\rightarrow 2^{10}$ Bits ; MEGA $\rightarrow 2^{20}$ Bits ; GIGA $\rightarrow 2^{30}$ Bits.

* Requirement is 8 KB

B : Bytes

8 bits make a Byte

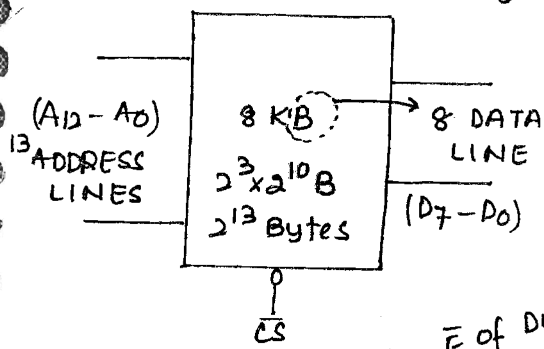
memory location $\rightarrow$ 8 K B $\leftarrow$ 8 bits / location

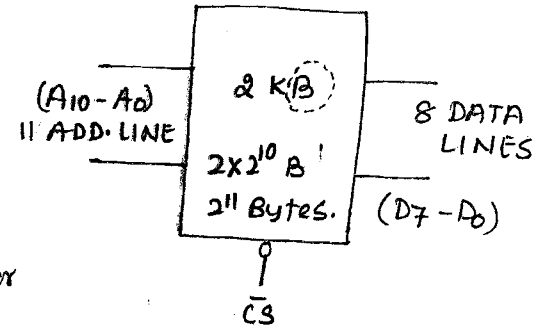
$$8K \rightarrow 2^3 \times 2^{10}$$
$$= 2^{13} = 2^m.$$

$\boxed{m = 13}$ $\leftarrow$ Address lines.

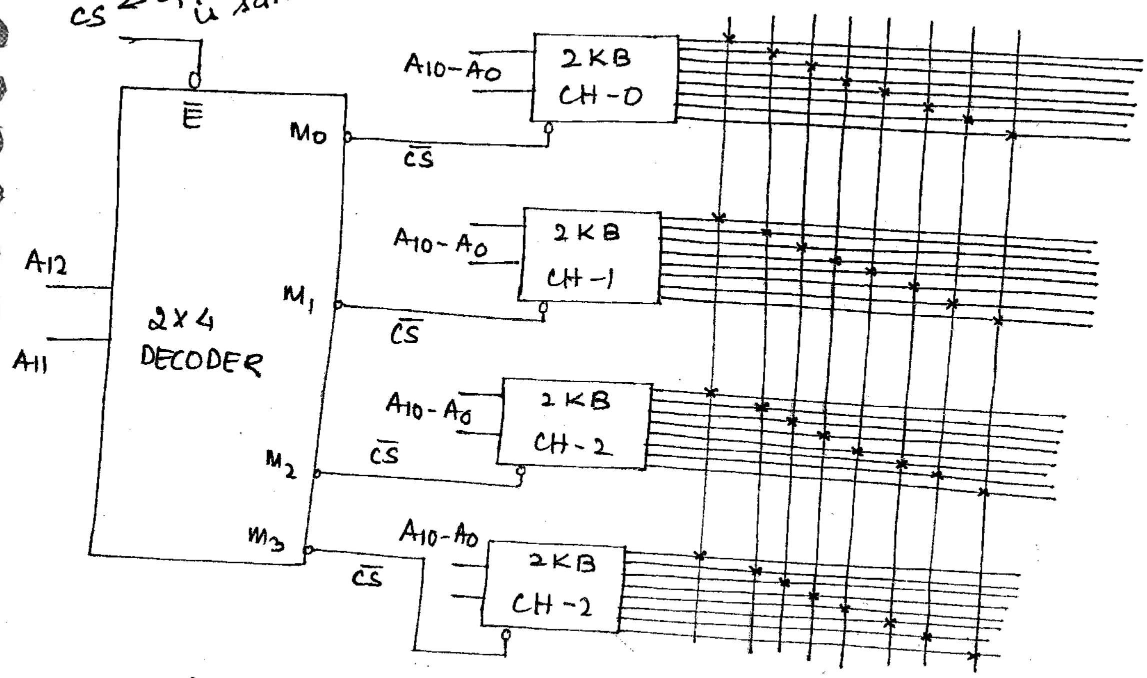
$\boxed{\text{data lines} = 8}$

B → Bytes ie 8 bits





$\overline{CS}$ ← of 8 KB RAM is same as $\overline{CS}$ or $\overline{E}$ of Decoder



<u>Note</u>:

A_{12}	A_{11}	$\underbrace{A_{10}\ A_9\ A_8\ A_7\ A_6\ A_5\ A_4\ A_3\ A_2\ A_1\ A_0}_{\text{2KB RAM}}$
0	0	CHIP 0
0	1	CHIP 1
1	0	CHIP 2
1	1	CHIP 3

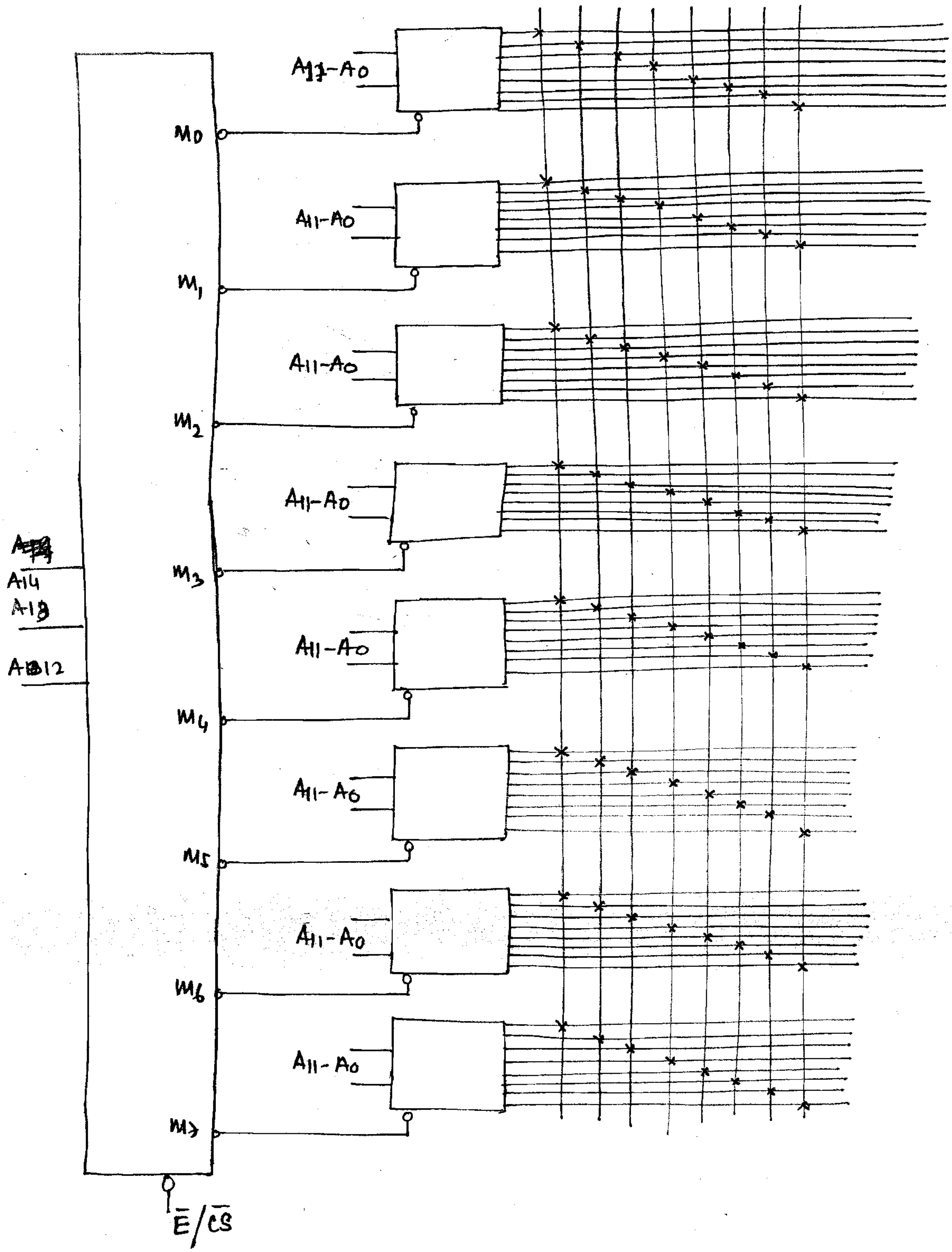
Q2) construct 32 KB RAM using 4 KB ROM.

Soln: 32 KB ROM   4 KB ROM

$2^5 \times 2^{10}$ Bytes $2^2 \times 2^{10}$ Bytes

Address lines = 15   Address line = 12.

Data line = 8   Data line = 8.



A14
A13
A12
M0
M1
M2
M3
M4
M5
M6
M7
A11-A0
A11-A0
A11-A0
A11-A0
A11-A0
A11-A0
A11-A0
A11-A0
$\overline{E}/\overline{CS}$

* Content :-

**1) Basics
2) Steady state AC circuits (Resonance)
3) Network Theorems
**4) Transient Analysis ← Very Important
**5) Two Port Network
6) Filters
7) Magnetic coupled circuits
8) Graph Theory

only memory Based Questions are asked. Don't waste much time on Revision.

* Books :-

1) Fundamentals of Electric circuits - Alexander & Sadiku.
2) Engg. ckt Analysis - Hayt & Kemmerly
3) Network Analysis - Van Valkenburg
(Transients & Two Port)
↳ In conventional.

* IAS → obj (Made Easy book)
→ conventional

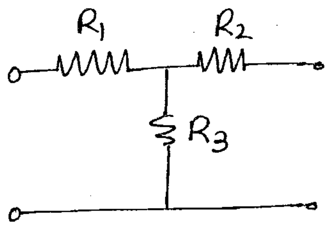
* Home Work
* Work Book
* Previous Papers (Gate) → EC. → EE. → IN.

IES → EC → EE
→ Memory
→ Problems (Num + Theory)

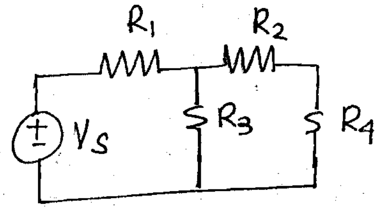
* Previous PSU papers.

* Test Series → old. → new.

*Network & circuits :-



T Network
↳ Combⁿ of element
↳ may or may not be closed



N/w or circuit
↳ Combⁿ of element
↳ necessary condⁿ is closed path.

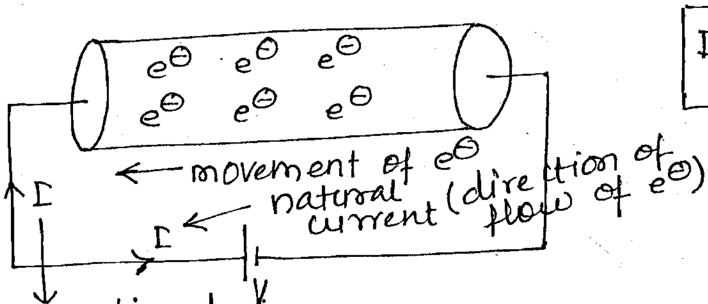
*All circuits are considered as networks but all networks cannot be considered as circuits.

* Network is a combⁿ of elements, it may or may not consist of closed path.

* circuit is also a combⁿ of elements and it should consist of closed path.

*charge (Q), I, V, P, W :-

$$\boxed{q = -1.602 \times 10^{-19} c}$$



$$\boxed{I = \frac{dq}{dt}}$$ → unit is coulomb/sec or Ampere.

Mag. of Conventional Current	= Mag. of Natural current

Conventional current (opposite to the flow of e⊖)

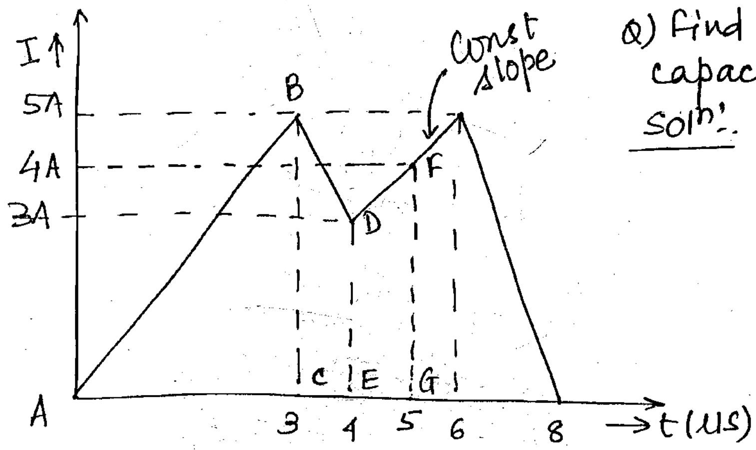
$$\boxed{I = \frac{dQ}{dt} \qquad Q = \int_{-\infty}^{t} I\,dt}$$

$$Q = \int_{-\infty}^{t} I\,dt = \int_{-\infty}^{0} I\,dt + \int_{0}^{t} I\,dt$$

$$\boxed{Q = Q_0 + \int_{0}^{t} I\,dt}$$

Q_0 → Initial charge

Note :-
* In circuit Theory we only consider the "conventional current" and not the "Natural current"
* KCL and KVL are based on the "CONVENTIONAL CURRENT"



Q) Find charge aquired by the capacitor in 5μs

Soln: 0-3μs (Region ABC)

$Q = \int I dt$ = Area under current time curve

$= \frac{1}{2} \times 3 \times 5 = 7.5$

(3μs-4μs) (Region BCDE)

$\Rightarrow$ Trapezoidal shape

$= \frac{1}{2}$ (sum of two heights) x (distance b/w two heights)

$= \frac{1}{2} \times (5+3) \times 1 = 4$

(4μs-5μs) (Region DFGE)

$\Rightarrow$ Trapezoidal shape

$= \frac{1}{2} \times (3+4) \times 1 = 3.5$

So total Area $= 7.5 + 4 + 3.5$

$= 15\,\mu C$

* To move an $e^{\ominus}$ from one place to another we require an external force called as EMF. ("Electromotive force") So, mathematically

$$\boxed{V = \frac{dW}{dQ} \text{ (Joules/C) or Volts}}$$

* Time Rate of change of work is called Power. Mathematically,

$$P = \frac{dW}{dt} \text{ (Joules/sec) or Watt}$$

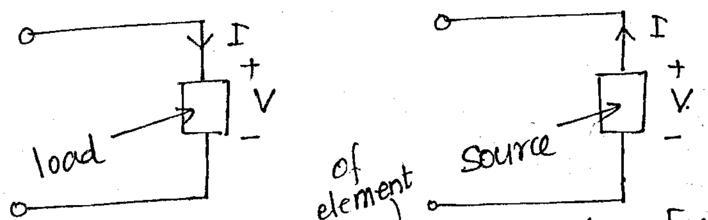
$$\Rightarrow P = \frac{dW}{dQ} \times \frac{dQ}{dt}$$

$$P = VI = I^2R = \frac{V^2}{R}$$

Also, $\boxed{G = \frac{1}{R} = \text{Conductance}}$

Hence, $P = I^2/G = V^2G$

*Note :



* current entering at the +ve terminal of the element
* Absorbing Power
* Acts as load.

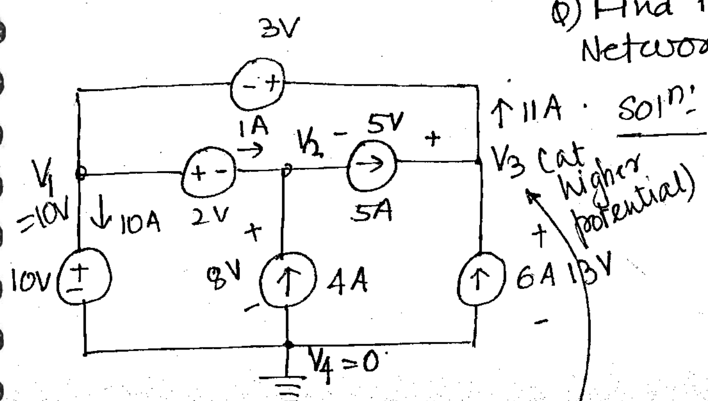
of element

* Current entering at the -ve terminal of the element — or current leaving from the +ve terminal of the Element — of element
* Delivering Power
* Acts as Source

* When the current is Entering at the "+ve terminal", the element is "Absorbing Power"

* when the current is leaving from the "+ve terminal" the element is "Delivering Power".

Q) Find Power of each element of the Network shown:.



$$\boxed{\begin{aligned} V_1 - V_2 &= 2 \\ \Rightarrow V_2 &= V_1 - 2 = 8V \end{aligned}}$$

$$\boxed{\begin{aligned} V_3 - V_1 &= 3V \\ V_3 &= 3 + V_1 = 13V. \end{aligned}}$$

Soln:. $P_{10} = 10V \times 10A = 100$ watts (Absor)

$P_4 = (-2+10) \times 4 = 32$ watts (delivering)

$P_5 = (-2-3) \times 5A = -25$ watts. (delivering)

$P_6 = (+3+10) \times 6A = 78$ watts (delivering) ~~absorbing~~).

$P_2 = 2V \times 1A = 2W$ (absorbing)

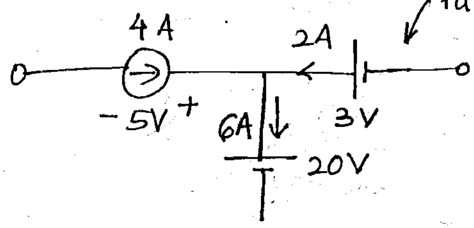
$P_3 = 11A \times 3V = 33$ watts (absorbing)

$P_4 = 4A \times 8V = 32$ watts.
$P_6 = 13V \times 6A = 78$ watts.
$P_5 = 5A \times 5V = 25$ watts.
} Delivering Power

$P_{10} = 10V \times 10A = 100$ watts
$P_2 = 2V \times 1A = 2$ watts
$P_3 = 11A \times 3V = 33$ watts
} Absorbing Power.

*Note:

$$\boxed{(P_T)_{absorb} = (P_T)_{delivered.}}$$ ← Satisfies for all networks.



Part of Network given.

Q) Find total power absorb... of fig. shown:.

soln:. $P_0 = 20V \times 6A$
$= 120$ watts (A)

$P_4 = 5V \times 4A = 20$ watts (D)

$P_3 = 3V \times 2A = 6$ watts. (D)

also, $P_4 = -20$ watts (Absorbing)
$P_3 = -6$ watts (Absorbing).

so, total power absorbing $= 120 - 20 - 6 = 94$ watts (Absorbing)

* NOTE:.
* when only any part of Network is given we have to follow above steps to calculate total Absorbing or Delivering power.

* Power is always positive, in real time power is never considered to be as -ve and the same is valid for Voltage also. For eg

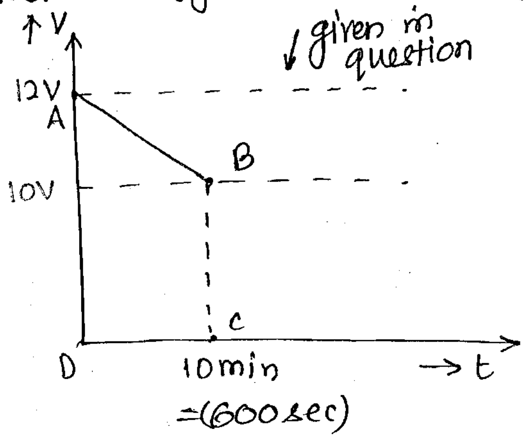
Bulb → 40W (we donot say -40watt Bulb since it is absorbing Power)

Battery → +12 V (we donot say -12 V Battery which is source and it delivers power)

* Energy:.
* capacity to do any work is called as Energy

$$\boxed{W = \int_0^t P\,dt}$$ → unit watt-sec or Joules.

Q) A fully charged mobile phone is good for 10 min talk time. During talk time battery delivers a const. current of 2A. Find Energy of the Battery during talk time?



Soln: * calculations for energy, time should always be in seconds.

Area of ABCD = $\frac{1}{2}$ × (sum of 2 heights × (dist. b/w 2 heights)

$= \frac{1}{2} \times (12+10) \times 600$

$\boxed{V \times t = 6600}$

So, $\boxed{W = VIt} \Rightarrow \boxed{W = P \times t}$

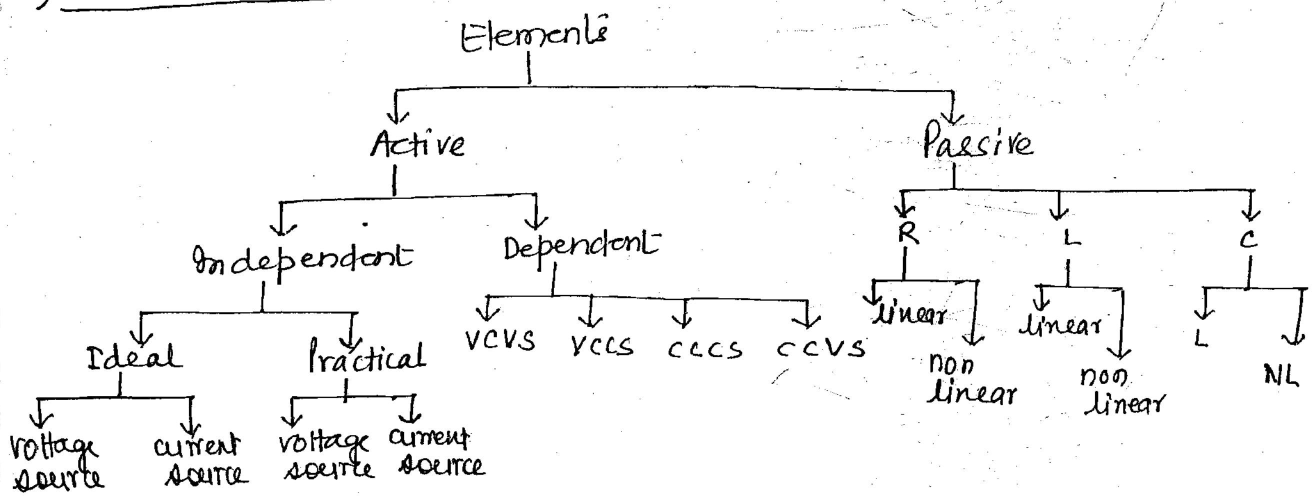
$W = 6600 \times 2A$

$\boxed{W = 13.2\ KJ}$

* CLASSIFICATION OF ELEMENTS:

1) Active & Passive elements
2) Linear & Non linear elements.
3) Unidirectional & Bi directional elements
4) Time Variant & Invariant elements
5) Lumped & Distributed elements.

1) Active & Passive Elements:



*ACTIVE ELEMENT :

* When the Element is capable of Delivering Energy Independently for long time (approx infinite time), then "ACTIVE ELEMENT"

OR

when the Element is having property of Internal amplification then it is called as "ACTIVE ELEMENT"

* Examples :
  1) Voltage source.
  2) current source. } Independent sources.
  3) Transistor, &
  4) OP-AMP } Dependent sources.

Note :
* when the 'C' is connected to DC, the capacitor is charging and while discharging it delivers energy independently, and that energy delivered to the ckt depends on the time constant of the ckt, whereas the ACTIVE ELEMENT delivers energy Independently.
* During discharging capacitor can deliver energy independently for short time (depends on its time const) and capacitor is not having the property of Internal Amplification. Hence capacitor is not an "ACTIVE ELEMENT".

* PASSIVE ELEMENT :

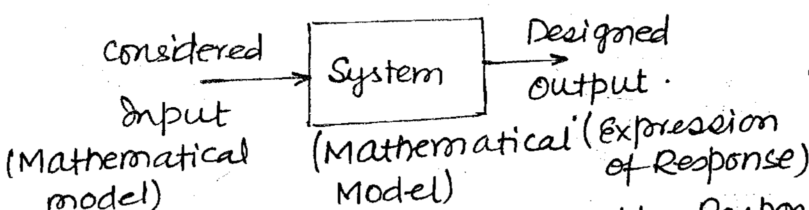
* when the Element is not capable of delivering energy Independently then it is called as "PASSIVE ELEMENT"

* Examples :
  1) Resistor
  2) Bulb
  3) Transformer ** $\boxed{V_1 I_1 = V_2 I_2}$ → *** $\boxed{\text{Internal power} = \text{External power.}}$
     ↳ step up or step down the voltage, but no power is stepped up or stepped down Hence no Internal amplification

*WHY SIGNALS AND SYSTEMS:-

* To ensure suitable working of the system to be designed before its actual designing. This is done by providing a signal to ensure the response.
* And by mathematical tool these can be done.
* Considering the system as mathematical Model and also considering the input as mathematical. The desired system can be designed.

* To find the expression of the Response we study signal & System.



* Mathematical tools used for find the Response of the system in more efficient way with less effort are:-

i) Fourier Series.
ii) Fourier Transforms.
iii) Laplace Transforms.
iv) Z Transforms.

} used to minimize the effort in designing of the system.

Note:-

* Information (Signal) can exist in only two ways:-

i) Continuous Time signal.
ii) Discrete Time Signal. (if samples are taken at very close intervals then only information can be Retrieved back).

* Sampling Theorem provides guidelines to convert Continuous time signals into equivalent Discrete Time Signals.

SIGNALS :.

* Any entity having associated information with it is defined as SIGNAL.
* Signal here means Voltage and current signals where both are functions of time.

$$\left.\begin{array}{c} v(t) \\ \text{or} \\ i(t) \end{array}\right\} f(t) \leftarrow \text{1 DIMENSIONAL SIGNAL}$$

* Signals need not always be function of time.
* Signals also can be function of space having different intensity level.

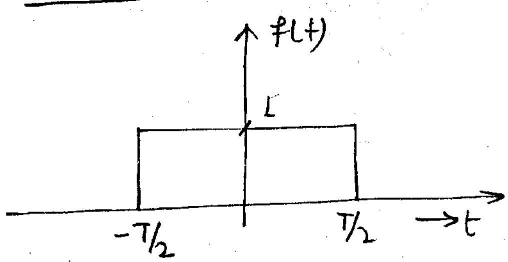
signal independent of time.

$\boxed{\text{Photo Picture} \rightarrow f(x, y).}$ ← 2 DIMENSIONAL SIGNALS

* Also the moving picture (Video signal) which is made up of various still frame is also a signal which is func^n of space & time.

Hence $\boxed{\text{Video} = f(\underbrace{x, y, t}_{\text{space, time}}).}$ ← 3 DIMENSIONAL SIGNAL

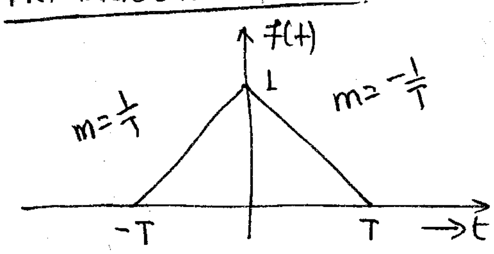
* A signal may be function of n variable. These signals are called as N DIMENSIONAL SIGNALS. ↳ need not always be time always.
* Signals can be represented mathematically or graphically. Analysis of signals can be done easily when graphical format is considered.
* RECTANGULAR PULSE :.



$$\boxed{f(t) = \begin{cases} 1 ; & -T/2 \le t \le T/2 \\ 0 ; & \text{otherwise.} \end{cases}}$$

* Any signal having short duration or existing for short duration is called a Pulse.

* TRIANGULAR PULSE :.



$f(t) = f_1(t) = m_1 t + C_1$

$$\boxed{f_1(t) = \frac{1}{T} t + 1 \; ; \; -T \le t \le 0}$$

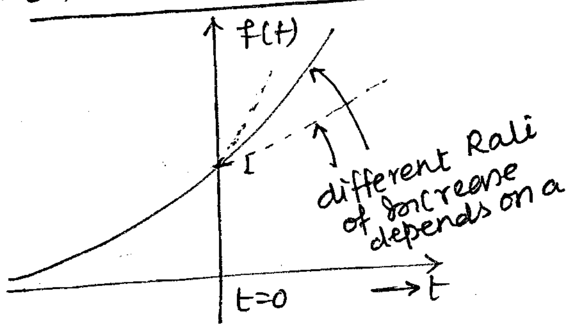
$f(t) = f_2(t) = m_2(t) + C_2$

$$\boxed{f_2(t) = \frac{-1}{T} t + 1 \quad 0 \le t \le T}$$

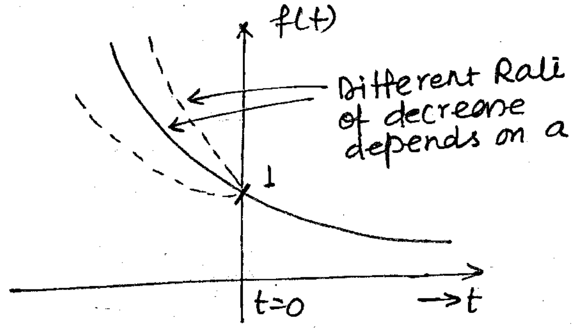
So, for Triangular Pulse:.

$$f(t) = \begin{cases} f_1(t) = \frac{1}{T}t + 1 & ; -T \leq t \leq 0. \\ f_2(t) = -\frac{1}{T}t + 1 & ; 0 \leq t \leq T \\ 0 & ; \text{otherwise}. \end{cases}$$

* EXPONENTIAL SIGNALS:.



$f(t) = e^{at}$

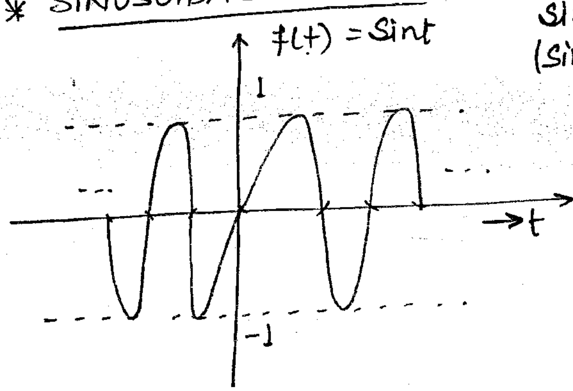


$f(t) = e^{-at}$

$\boxed{a = \text{Scaling factor}}$ (deciding Rate of Increase or Decrease).

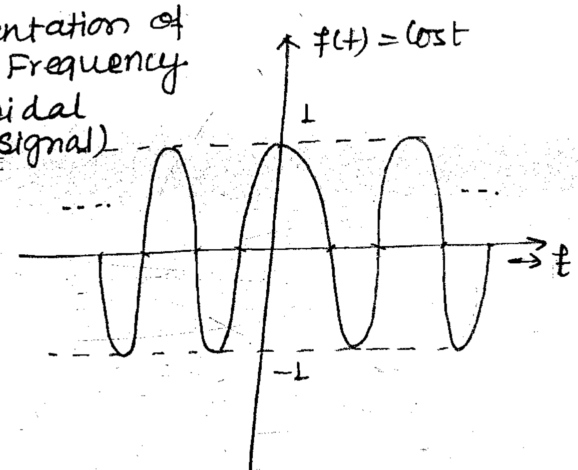
* a is also called as the Time Const as they decide Rate of Rise and decrease.

* SINUSOIDAL SIGNALS:. * Representation of Single Frequency (Sinusoidal signal)



* Zero cross over are:.

$\boxed{t = \pm n\pi}$

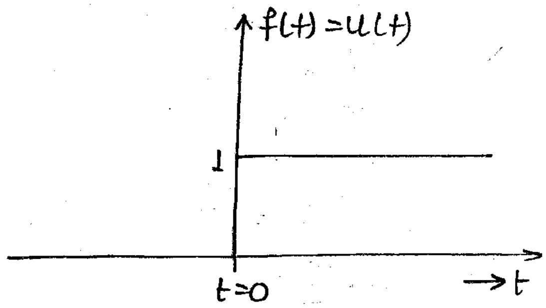


* Zero cross overs are:.

$\boxed{t = \pm (2n+1)\pi/2}$

* The instance of time where signals oscillating b/w +ve and -ve values cross 0 value are defined as ZERO CROSS OVER of such oscillating signals.

* UNIT STEP SIGNAL:



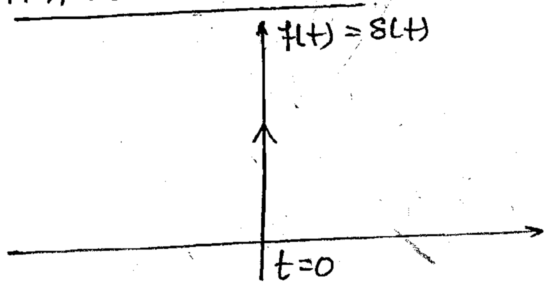
$$\boxed{f(t) = \begin{cases} 1 ; & t > 0 \\ 0 ; & t < 0 \end{cases}}$$

$$\boxed{f(t) = u(t) = \begin{cases} 1 ; & t \geq 0 \\ 0 ; & t < 0 \end{cases}}$$

COMPROMISED DEFINITION

$u(t) = 1 ; t = 0$.

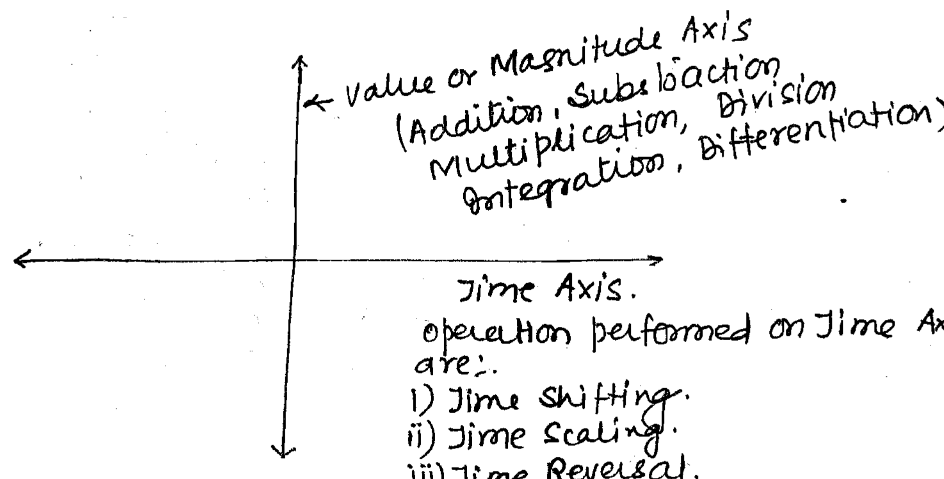
* IMPULSE FUNCTION:



$$\boxed{\begin{aligned} f(t) = \delta(t) &= 0 ; \; t \neq 0 \\ &\neq 0 ; \; t = 0 \\ \int_{-\infty}^{\infty} \delta(t)\,dt &= 1 \end{aligned}}$$

* Impulse impacts can be measurable or unmeasurable. Analysis is done only for measurable impacts.
* Hence to analyse the impulse signal it has to be measurable and for that its Area should be equal to Unity.
* The magnitude of $\delta(t)$ is ∞ at $t=0$ and hence unmanagaeble so to manage them indirectly its Area is made equal to 1.

Note :



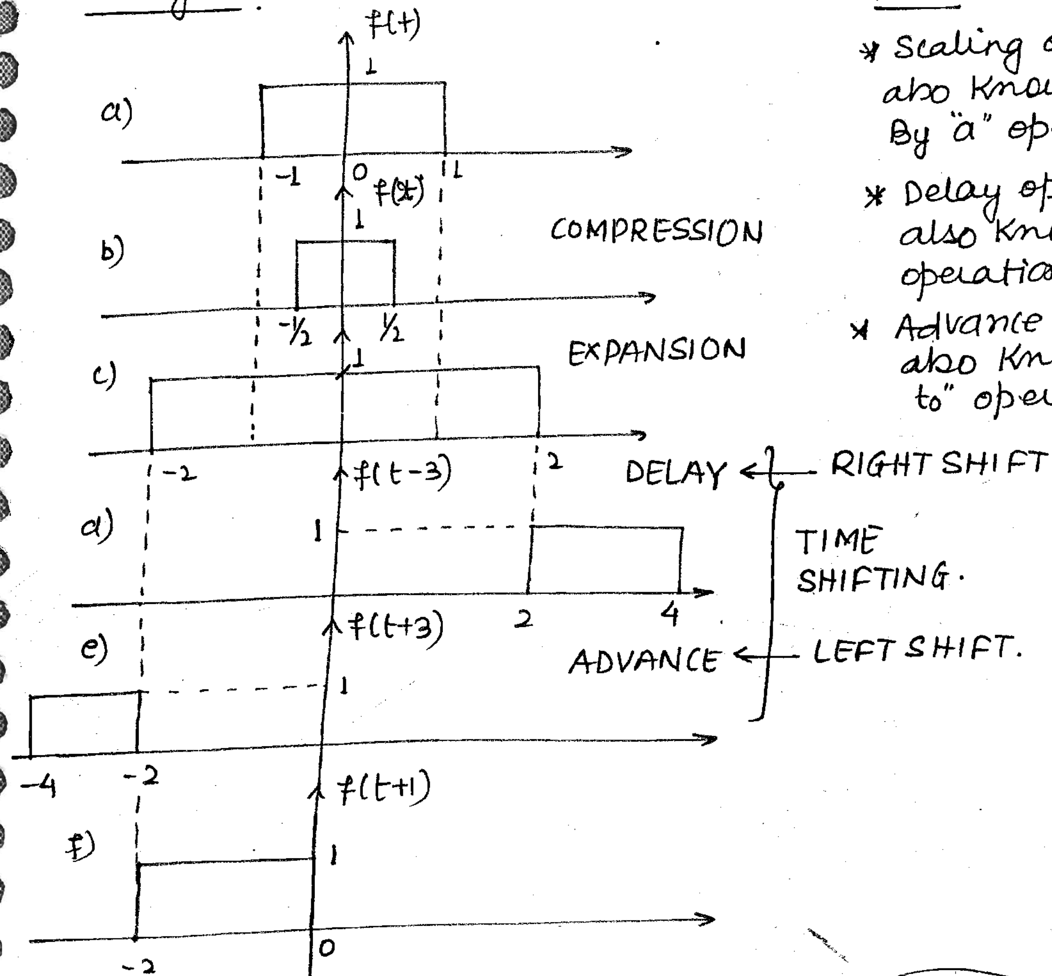
operation performed on Time Axis are:

1) Time Shifting.
ii) Time Scaling.
iii) Time Reversal.

*Analysis :

*Note :

* Scaling operation is also known as DIVIDE By "a" operation.
* Delay operation is also known as ADD "to" operation.
* Advance operation is also known as "SUBTRAC to" operation.



a) $f(t) = 1 \;;\; -1 \le t \le 1$
$\qquad\;\; 0 \;;\;$ otherwise

b) $f(2t) = 1 \;;\; -1 \le 2t \le 1$
$\qquad\;\; 0 \;;\;$ otherwise

$$\boxed{\begin{aligned} f(2t) &= 1 \;;\; -\tfrac{1}{2} \le t \le \tfrac{1}{2} \\ &\;\; 0 \;;\; \text{otherwise.} \end{aligned}}$$

c) $f(\tfrac{1}{2}t) = 1 \;;\; -1 \le t/2 \le 1$
$\qquad\;\; 0 \;;\;$ otherwise

$$\boxed{\begin{aligned} f(t/2) &= 1 \;;\; -2 \le t \le 2 \\ &\;\; 0 \;;\; \text{otherwise.} \end{aligned}}$$

Note :

$$\boxed{\begin{aligned} &t \to at \\ &f(t) \xrightarrow{t \to at} f(at) \end{aligned}}$$

$$\boxed{\begin{aligned} &a > 1 \to \text{COMPRESSION} \\ &a < 1 \to \text{EXPANSION} \end{aligned}}$$

$$\boxed{a = \text{SCALING FACTOR.}}$$

d) $f(t-3) = 1 ; \quad -1 \le t-3 \le 1$
$\quad = 0 ;$ otherwise

$$\boxed{\begin{aligned} f(t-3) &= 1 ; \quad 2 \le t \le 4 \\ &\quad 0 ; \quad \text{otherwise} \end{aligned}}$$

e) $f(t+3) = 1 ; \quad -1 \le t+3 \le 1$
$\quad 0 ;$ otherwise

$$\boxed{\begin{aligned} f(t+3) &= 1 ; \quad -4 \le t \le -2 \\ &\quad 0 ; \quad \text{otherwise} \end{aligned}}$$

f) $f(t+1) = 1 ; \quad -1 \le t+1 \le 1$
$\quad = 0 ;$ otherwise

$$\boxed{\begin{aligned} f(t+1) &= 1 ; \quad -2 \le t \le 0 \\ &\quad 0 ; \quad \text{otherwise} \end{aligned}}$$

*Note :.

i) $t \rightarrow t - t_0 \rightarrow$ delay or Right shift

ii) $t \rightarrow t + t_0 \rightarrow$ Advance or left shift.

(3) Nadir
(Antonym)
(opposite word)

a) highest ✓
b) lowest
c) medium
d) integration.

zenith → the topmost point.
Nadir → The lowest point.

(4) odd one out:

zenith
pinnacle
Apotheosis
Nadir ✓ odd.

} The highest point

- summit
- Peak
- Apex
- Acme
- culmination.

The/Theo : God

- Thiest → one who believes in god.
- Athiest → one who does not believe in god.
- Athiest → one who does not believe the existence of god.
- Agnostic → one who is doubtful / sceptical about the existence of god.
- Theocracy → Rule by Religion (Vatican city Ex.)
  cracy → Rule.
- Paparchy → Rule by church.

(5) Nocturnal : Bat

✓ a) Amphibian : Frog.
b) sly : cat.
c) carnivorous : cow
d) Aquatic : Liz. (Lizard)

Amphibian → stay on both land & water.

carnivourous → Flesh eating.

sly → cunning.
→ deceitful.
→ cheat
→ shrewd.

clever → Astute, sagacious. (wise)

sage → one who renounces worldy pleasures.

carni : Flesh

Incarnation → To take rebirth.
to take bodyily form.

vor : eating

- varacious → Greedy / never satisfied.
- Verocious → Throuthful, honest

ver : truth

verify →

Chapter 1
conjunctions

conjunctions are joining words.

conjunction is a word that joins two or more words, phrases or clauses.

① she could not come to the party as she was ill

conjunction.

2 clauses

• clause → clause is any part of sentence which has its own subject and verb.

clause gives complete meaning

② People's ignorance and population explosion are two interrelated problems. 2 phrase.

conjunction.

phrase is just a collection of words. (शब्दसमूह). It does not have complete meaning.

③ Ravi and Amit are good friends.

conjun

2 words

• Use of conjunctions:

(1) Both ① Both is followed by 'and'.

② Both-and takes plural ~~word~~ verb and pronoun.

• Both is not used in negative sentences.

• In negative sentences both-and is replaced by neither-nor and both of is replaced by neither of.

• Neither nor follows the rule of proximity (closeness) to decide the verb. • Neither of takes plural noun & singular verb.

• Neither of ~~are~~ always takes singular verb.

① Both Ravi and Amit are sincere. ✓

✗ • Both Ravi and Amit are not doing their work efficiently

✓ → Neither Ravi nor Amit is doing his work efficiently.

* NUMBER SYSTEM :-

V → Viniculum (BAR)
B → Bracket. {}
O → of.
D → Division. (÷)
M → Multiplication (x)
A → Addition. (+)
S → substraction (-)

Q1) Convert the following recurring terms into their corresponding p/q forms?

a) $27.\overline{17}$ ↓ Complete Bar $27.171717...$ (Bar immediately after point).

b) $27.2\overline{17}$ ↓ Partial Bar $27.21717...$

c) $0.00\overline{17}$ ↓ Partial Bar $0.00171717....$

Soln:. a) $27.\overline{17}$

$x = 27.171717...$

$100x = 2717.1717....$

$100x - x = \cancel{27}\ 2717.1717 - 27.1717$

$99x = 2690$

$$\boxed{x = \frac{2690}{99}}$$

SHORTCUT

$x = 27.\overline{17}$

$p/q = \dfrac{2717 - 27}{\underline{99}}$

$= \dfrac{2690}{\underline{99}}$

b) $27.2\overline{17}$

$x = 27.21717...$

$\dfrac{p}{q} = \dfrac{27217 - 272}{\underline{9}\,\underline{9}\,\underline{0}}$

$$\boxed{\frac{p}{q} = \frac{26945}{990}}$$

c) $0.00\overline{17}$

$\dfrac{p}{q} = \dfrac{00017 - 000}{9900}$

$$\boxed{\frac{p}{q} = \frac{17}{9900}}$$

Q2) $27.\overline{27} \times 33 + 6$

Soln:. $\frac{(2727 - 27)}{\cancel{99}\,3} \times \cancel{33} + 6$

$= \frac{2700 \times \cancel{3}}{3} + 6$

$= 906.$

Q3) What is the unit's digit in the expansion of a) $(766)^{136}$.

b) $(277)^{134}$.

c) $(454)^{41}$.

d) $(888)^{103}$.

e) $(1028)^{100}$.

f) $(459)^{40}$.

Soln:. a) $(766)^{136}$ ← Based on cyclicity or Power cycle

$0^N = 0$

$1^N = 1$

$2^N =$

$2^1 = 2$

$2^2 = 4$

$2^3 = 8$ → cyclicity of 2^N is 4 ie 2, 4, 8, 6

$2^4 = 16$

$2^5 = 32$

$2^6 = 64$

$2^7 = 128$

$2^8 = 256$

$2^9 = 512$

3^N ← cyclicity is 4 ie (3, 9, 7, 1)

$3^1 = 3$

$3^2 = 9$

$3^3 = 27$

$3^4 = 81$

$3^5 = 243$

$3^6 = 729$

3^7

4^N ← cyclicity is (4, 6)

$4^1 = 4$

$4^2 = 16$

$4^3 = 64$

$4^4 = 256$

$4^5 = 1024$

NUMBERS	FREQN OF NOS. as POWER CYCLE
0, 1, 5, 6	STAY AS IT IS
2, 3, 7, 8	4
4, 9	2

a) $\cancel{(766)^{136} \longrightarrow \text{unit digit} = 6}$.

b) $\cancel{(277)^{134} \longrightarrow 4}$

c) $\cancel{(454)^{\cancel{134}\,41} \longrightarrow 2}$

d) $\cancel{(888)^{103} \longrightarrow 4}$

a) $(766)^{136} = 766 \times 766 \times 766 \times 766 \times \cdots\cdots$ 136 times.

$= \ldots 6 \times \ldots 6 \times \ldots 6 \times \ldots 6 \times \ldots$

$= \ldots\ldots 6$

$$\boxed{(\ldots\ldots 0/1/5/6)^{\alpha\alpha\alpha\alpha\ldots\alpha} = \ldots\ldots 0/1/5,6}$$

b) $(277)^{134} = 277 \times 277 \times 277 \times 277 \times \cdots\cdots$ 134 times.

$= \ldots 7 \times \ldots 7 \times \ldots 7 \times \ldots 7 \times \ldots\ldots$ 134 times.

$(\ldots\ldots) \times (\ldots\ldots) \times (\ldots\ldots) \times \ldots\ldots \times (277 \times 277)$

49

units digit is (9)

33 ← Complete sections.

$4 \,\overline{)\,134}$

12

14

12

×2

<u>Short cut</u>:

$(277)^{134} \longrightarrow$ Power cycle = 4

33

$4 \,\overline{)\,134}$

12

×14

12

×2

$7^{2} = 49$

units place (units digit)

* $(454)^{41} \longrightarrow$ Power cycle = ~~4~~2

$$2\,\overline{\smash{)}41}\quad 20$$
$$40$$
$$\times\, 1$$

$4^1 = 4$

* $(888)^{103} \longrightarrow$ Power cycle = 4

$$4\,\overline{\smash{)}103}\quad 25$$
$$100$$
$$3$$

$8^3 = 512$

* $(1028)^{100} \longrightarrow$ Power cycle = 4

$$4\,\overline{\smash{)}100}\quad 25$$
$$100$$
$$000 \leftarrow \text{Remainder}$$

X

$8^0 = 1$

* $(459)^{40} \longrightarrow$ Power cycle = 2

Remainder = 0

*Special case of Remainder zero:-

* All complete sections.
* No incomplete section.

$(1028)^{100} \longrightarrow$ P.C = 4

$$4\,\overline{\smash{)}100}\quad 25$$
$$100$$
$$xxx$$
$$0$$

$8^4 = 8^2 \times 8^2$
$= 64 \times 64$
$= ....6$

~~40~~ $(459)^{40} \longrightarrow$ P.C = 2

$$2\,\overline{\smash{)}40}\quad 20$$
$$40$$
$$00$$

$9^2 = 9 \times 9 = 81$

* Remainder 0 short cut:-

After getting Remainder = 0 → Even Base = 6; → odd Base = 1

Q4) What is the unit's digit in the expansion of the following expression:-

$$(666)^{666} \times (877)^{134} + (959)^{20}$$

$$4\,\overline{\smash{)}134}\quad 33$$
$$132$$
$$2$$

$7^2 = 49$

$$4\,\overline{\smash{)}20}\quad 5$$
$$20$$
$$00$$

$(959) \rightarrow$ odd Base $= 1$

Soln:- $(....6 \times .... 9) + ... + 1$
$= (.....4) + (....1) = ...5$

SUMMARY

UNIT'S DIGIT

- 0, 1, 5, 6 (STAY AS IT IS NUMBER)
- 2, 3, 7, 8, 4, 9
  - NON ZERO REMAINDER → PROCEDURE
  - ZERO REMAINDER
    - PROCEDURE
    - S.C
      - BASE ODD (1)
      - BASE EVEN (6)

Q) How many zeroes are there at last in the expansion of

a) $25 \times 4 \times 8 \times 7 \times 10 \times 16 \times 100$

b) $(25)^{125} \times 4^{40}$

Soln:. a) $25 \times 4 \times 8 \times 7 \times 10 \times 16 \times 100$

$100 \times 56 \times 16 \times 1000$

$= 56 \times 16 \times 100000$

$5^2 \times 2^2 \times 2^3 \times 7 \times 5 \times 2 \times 2^4 \times 2^2 \times 5^2$

$= 2^{12} \times 5^5 = 2^7 \times 2^5 \times 5^5$ least power

LEAST POWER $= 2^7 \times 10^5$

$=$ 5 ZEROS

b) $(25)^{125} \times (4)^{40}$

$(5)^{250} \times 2^{80}$ LEAST POWER

80 ZERO

Note:. $7^{125} \times 4^{50}$

$7^{125} \times 2^{100}$

NO ZEROS

ZERO'S AT LAST CONDITION :.

i) Multiple of 10. → direct multiple ie 10, 100, 1000 - - - -

ii) Hidden multiple → (2, 5)

*The total no. of (2×5) combinations = no. of zeros at last in the expansion

$$\boxed{\text{(total no. of } (2\times5) \text{ combn)} = \text{(no. of zeroes at last in expansion)}}$$

Q6) How many zeros are there at last in the expansion of :.

a) 6!
b) 10!
c) 100!
d) 145!
e) 1000!

Note :.
1! ; 2! = 2 ; 3! = 6; 4! = 24
$\boxed{5! = 120}$
↓ onwards only zeros will start coming not before that.

Soln :. a) 6! = 6x5x4x3x2x1
= $6 \times 5^1 \times 3 \times 2^3$
= 1 ZERO.
720

b) 10! = 10x9x8x7x6x5x4x3x2x1
= $2 \times 5 \times 9 \times 2^3 \times 7 \times 3 \times 2 \times 5 \times 2^2 \times 3 \times 2 \times 1$
= $2^8 \times 5^2$
= 2 ZEROS
(3628800)

c) 100!
↓

100 → 20x5	65 → 13x5	30 → 6x5
95 → 19x5	60 → 12x5	25 → 5x5
90 → 18x5	55 → 11x5	20 → 5x4
85 → 17x5	50 → 10x5 = 2x5x5	15 → 3x5
80 → 16x5	45 → 9x5	10 → 2x5
75 → 15x5 = 3x5x5	40 → 8x5	5 → 1x5
70 → 14x5	35 → 7x5	

Note :. For 100! Zeroes are by default. They will come by default. and no. of zeroes depends on no. of 5's present in it.

100! = 1x2x3x4x5 | x6x7x8x9x10 | x11x12x13x14x15 | x16x17x18x19x20 | x ---- x ---- x100
(2x5, 3x5, 4x5)

$\frac{100}{5}$ = 20 sections ← divide the complete 100! in 20 sections.

* In these sections some special nos (which contain 2 5's will also be there). such as :.

Already taken into account in dividing sections.
25 → 5 x 5 (NOT TAKEN INTO ACCOUNT in 20 SECTIONS)
50 → 5 x 5 x 2
75 → 5 x 5 x 3
100 → 5 x 5 x 4

→ NOW taking = $\left(\frac{100}{5} + \frac{100}{25}\right) = 24$.

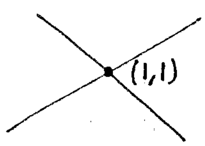
*LINEAR ALGEBRA :-

Analysis

$x+2y=3$
$2x+3y=5$

So, $x=1, y=1$

Intersecting Line



(x' and y')

$x+2y=3$
$2x+4y=6$

let $y=K$
$x=3-2K$

Infinite no. of solutions

COINCIDENT LINE

$x+2y=3$
$x+2y=5$

NO solution

(PARALLEL LINES)

*Any 1st degree 2 dimensional equation in $x+y$ represents a line in the XY PLANE. (LINEAR SYSTEM OF EQUATION IN 2 VARIABLES)

Note :-

*The study of LINEAR SYSTEM OF EQUATIONS is called LINEAR ALGEBRA.

$x+2y=3$
$2x+3y=5$

on solving the equation

$x=1; y=1$

(UNIQUE SOLUTION)

$x+2y=3$
$2x+4y=6$

let $y=K$
$x=3-2K$

(INFINITE NO. OF SOLUTION)

$x+2y=3$
$x+2y=5$

(NO SOLUTION)

*To study about the Linear system of equations, we require the concept "RANK OF MATRIX". Hence we study about MATRICES in the concept LINEAR ALGEBRA.

*MATRIX :-

*Arrangement of elements or numbers in Rows and Columns such that each row will have same no. of element and each column will have same no. of element is called a MATRIX.

*Operation on Matrixes :

1) Addition
2) Subsbraction
3) Multiplication $\{A_{m \times l} \times B_{l \times n} = C_{m \times n}\}$
4) TRACE OF SQUARE MATRIX :

*The sum of the PRINCIPAL DIAGONAL ELEMENTS OF A SQUARE MATRIX is called TRACE.

5) SYMMETRIC MATRIX :

When $\boxed{A^T = A}$ $\begin{bmatrix} 1 & 5 & -1 \\ 5 & 2 & 9 \\ -1 & 9 & 3 \end{bmatrix}$

the matrix A is ~~skew~~ symmetric

6) SKEW SYMMETRIC MATRIX :

COMPULSORY CONDITION (diagonal elements should be zero)

When $\boxed{A^T = -A}$. $\begin{bmatrix} 0 & 3 & -5 \\ -3 & 0 & 9 \\ 5 & -9 & 0 \end{bmatrix}$

then Matrix A is SKEW SYMMETRIC.

*DETERMINANT OF SQUARE MATRIX :

*For a 1x1 MATRIX, the no. ~~itsey~~ itsey is the Determinant

*For a 2x2 MATRIX of the form :

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

the determinant is given by. $(ad-bc)$

*MINOR OF AN ELEMENT :

let

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

then $\text{Minor of } a_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} = (a_{22}a_{33} - a_{32}a_{23})$

$$\text{Minor of } a_{21} = \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} = (a_{12}a_{33} - a_{32}a_{13})$$

*COFACTOR of an Element :-

* Minor of a_{ij} is M_{ij} ; then cofactor of a_{ij} is

$$\boxed{\text{cofactor of } a_{ij} = (-1)^{i+j} . M_{ij}}$$

* The Determinant of square matrix is defined as " The sum of product of elements of any row or any column with the corresponding cofactors"

* Analysis :-

* we have to find the determinant of given 4x4 matrix. For this choose any row or column having the max^m no. of zeroes.

let

$$A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 1 & 0 & 1 & -1 \\ 1 & 2 & 3 & 1 \\ 1 & 0 & 2 & 0 \end{bmatrix}$$

using 2nd column weget :-

$$2(-1)^{3+2}\begin{vmatrix} 1 & 2 & 1 \\ 1 & 1 & -1 \\ 1 & 2 & 0 \end{vmatrix}$$

$$-2\{1(0+2) - 2(0+1) + 1(2-1)\}$$

$$= -2$$

using 4th column weget

$$1.(-1)^{4+1}\begin{vmatrix} 0 & 2 & 1 \\ 0 & 1 & -1 \\ 2 & 3 & 1 \end{vmatrix} + 2(-1)^{4+3}\begin{vmatrix} 1 & 0 & 1 \\ 1 & 0 & -1 \\ 1 & 2 & 1 \end{vmatrix}$$

$$-1\{-2(0)+1(-2)\} - 2\{1(+2)+1(2)\}$$

$$+6-8$$

$$=-2$$

$$= -2$$

Note :-

* A matrix is said to be NON ~~SON~~ SINGULAR when

$$\boxed{DET(A) \neq 0}$$

and is said to be SINGULAR when

$$\boxed{DET(A) = 0}$$

* ** $\boxed{Det\,(AB) = (Det\,A)(Det\,B)}$

* ** Det (A+B) is not necessarily (Det A) + (Det B)

* ** If any two rows are same or constant multiples (columns) then Determinant of that Matrix is zero.

* ** If ~~some~~ SUM of the elements in every row or every column is zero then the determinant of such matrix is zero.

for eg.

$$\begin{bmatrix} 1 & 2 & -3 \\ 0 & 2 & -2 \\ 1 & 1 & -2 \end{bmatrix}$$ sum of Rows zero. (sum of each row is zero).

*ADJOINT OF SQUARE MATRIX :-

*It is the Transpose of cofactor Matrix ie

if $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

then the cofactor of $a_{ij} = A_{ij}$

then $\text{Adj } A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$

Note :-

* when ** $\boxed{A(\text{adj } A) = (\det A) I}$ $I \rightarrow$ Identity matrix.

* ** $\boxed{\det(\text{adj } A) = (\det A)^{n-1}}$; n= order of matrix

*) ** $\boxed{\text{Adj}(\text{adj } A) = (\det A)^{n-2} A}$

*INVERSE of SQUARE MATRIX :-

* A matrix B is said to be Inverse of a Non Singular matrix A if

** $\boxed{AB = BA = I}$

*To find A^{-1} we have

** $\boxed{A^{-1} = \frac{\text{Adj } A}{\text{Det } A}}$

*for Matrix A;

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

** $\boxed{A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \; ; \; \boxed{ad-bc \neq 0}}$

** * $\boxed{\det(A^{-1}) = \dfrac{1}{(\det A)}}$

*ELEMENTARY TRANSFORMATION ON A MATRIX :.

*There are only 3 elementary transformations; they are:.

✓1) Interchanging of any two rows ($R_1 \leftrightarrow R_2$)

✓2) Multiplication of a row by a constant ($R_2 \to 3R_2$)

✓3) Addition of 1 ROW to the corresponding elements of some other row ($R_2 \to R_2 + R_1$).

Note :.

$\left.\begin{array}{l} * R_2 \to R_2 + 3 \\ * R_2 \to R_2 \times R_1 \end{array}\right\}$ Not elementary xmation.

*Inverse of Matrix (using elementary xmation)

*GAUSS JORDAN METHOD :.

Q1) Find the Inverse of

$$A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

Soln:

Use this element to make all the elements below/above this as zero.

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 1 & 4 & 3 & 0 & 1 & 0 \\ 1 & 3 & 4 & 0 & 0 & 1 \end{array}\right]$$

$(R_2 \to R_2 - R_1)$; $[R_3 \to (R_3 - R_1)]$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

$(R_1 \to R_1 - 3R_2)$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 4 & -3 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

$R_1 \to R_1 - 3R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 7 & -3 & -3 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

Hence,

$$\boxed{A^{-1} = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}}$$

(Q2) Find the inverse of $A = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Soln:. By Gauss Jordan method:.

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 3 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$(R_1 \to R_1 - 3R_4)$; $(R_2 \to R_2 + 2R_4)$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

so, $\boxed{A^{-1} = \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$

*MINOR OF A MATRIX:.

* For finding the No. of minors of given order choose no. of rows or columns from given no. of Rows or Columns.

let $A = \begin{bmatrix} a_1 & b_1 & c_1 & d_1 & e_1 \\ a_2 & b_2 & c_3 & d_2 & e_2 \\ a_3 & b_3 & c_3 & d_3 & e_3 \\ a_4 & b_4 & c_4 & d_4 & e_4 \end{bmatrix}_{4\times5}$

Note:.

No. of minors of order 4 is 5. $(\underset{\text{choosing any Row.}}{{}^4C_4} \times \underset{\text{choosing any columns.}}{{}^5C_4} = 5)$

(4x4) ✓No. of minors of order 3 is ${}^4C_3 \times {}^5C_3 = 4\times10 = 40$ (choose any 3 rows or columns)

(3x3) ✓No. of minors of order 3 is ${}^4C_2 \times {}^5C_2 = 6\times10 = 60$ (choose any 2 rows or colums).

(2x2) ✓No. of minors of order 2 is $4\times5 = 20$.

(1x1) ✓No. of minors of order 1 is

*In general, for matrix $A_{m\times n}$:

i) ~~There~~ The no. of minors of order 'r' that can be generated is $({}^nC_r \times {}^mC_r)$.

ii) The order of greatest minor that can be obtained for this matrix is min (m,n).

$\begin{cases} A_{5\times2} \Rightarrow A_{2\times2} \to \text{greatest minor} \not\exists \text{ No}(A_{3\times3}). \\ A_{3\times7} \to A_{3\times3} \to \text{greatest minor} \not\exists \text{ No}(A_{4\times4}). \end{cases}$

RANK OF A MATRIX :.

*Exists for both Square as well as Rectangular matrix.

*A no. "r" is said to be the "RANK OF A MATRIX A" if :.

i) there exist a minor of order "r" of A which is not zero.

ii) all minors of order more than "r" of A must be zero.

for eg :.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 10 \end{bmatrix}$$

*All red dotted minors A > have det = 0.

*Green dotted minor donot have det = 0.

then $\det A = 0$

and $\det \begin{bmatrix} 4 & 6 \\ 6 & 10 \end{bmatrix} \neq 0 \Rightarrow 40-36 = 4$

*Note :. For given 3x3 matrix, the Minor of 3rd order is the given matrix itself. Also the det. of given minor is zero. Hence, also no. other minor of order 4x4 is available. Hence the matrix A cannot have $\rho(A)=3$. Hence we need to search for 2x2 minor and check for availability of such minor whose det $\neq 0$.

Hence, there exist a minor of order 2x2 whose det is not zero. Hence

$$\boxed{\begin{array}{l} \text{Rank} = 2 \\ \rho(A) = 2 \end{array}}$$

← Rank of Matrix can also be defined as the order of Largest non zero minor of the matrix (Here 2x2 minor).

Note :.

*To find the Rank of the matrix we can use ELEMENTARY XMATIONS.

*By converting, the given matrix into its "ECHELON FORM", "the no. of NON ZERO ROWS in the "ECHELON FORM IN THE MATRIX" represents the rank of the matrix."

Note :. Calculation of Rank through Minor calculation is very time taking. Hence we use Rank calculation through "ECHELON FORM."

*ECHELON FORM :.

*By applying Elementary Transformations we can convert a given matrix into a form in which :.

i) All zero Rows must be present below Non zero Rows.

ii) In the Non zero Rows; the no. of zeroes before the 1st non zero no. to the next row must increase.

*Such a Form is called "ECHELON FORM OF GIVEN MATRIX".

Q3) Find the Rank of :.

$$A = \begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

* Note :. Going through MINOR Calculations to obtain RANK OF MATRIX is time taking. Hence ECHELON FORM FORMATION is used to calculate the Rank of A MATRIX

$\boxed{\rho(A) = \text{RANK OF MATRIX } A}$

$$A = \begin{bmatrix} -2 & -1 & 3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

$R_2 \rightarrow 2R_2 + R_1$

$R_3 \rightarrow 2R_3 + R_1$

$$\begin{bmatrix} -2 & -1 & -3 & -1 \\ 0 & 3 & 3 & -3 \\ 0 & -1 & -1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

* No Zeroes before (−2)
* 1 zero before 3. Hence no. of zero increased from going from 1st row to 2nd row.
* 1 zero before (−1) hence no increase in no. of zero from 2nd to 3rd Row. Hence not in ECHELON FORM.

$R_3 \rightarrow 3R_3 + R_2$

$R_4 \rightarrow 3R_4 - R_2$

$$\begin{bmatrix} -2 & -1 & -3 & -1 \\ 0 & 3 & 3 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

All zero Row present below Non zero Row.

$$\boxed{\rho(A) = 2}$$

Note: (Assumption)

$$\begin{bmatrix} -2 & -1 & -3 & -1 \\ 0 & 3 & 3 & -3 \\ 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

matrix in Echelon form only.

$$\boxed{\rho(A) = 3}$$

Q4) Find the Rank of

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 & 9 \end{bmatrix}$$

Soln:

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 & 9 \end{bmatrix}$$

$R_2 \rightarrow 2R_2 - 3R_1$

$R_3 \rightarrow R_3 - 2R_1$

$R_4 \rightarrow 2R_4 - 5R_1$

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 & 6 \\ 0 & -1 & -2 & -3 & -4 \\ 0 & -1 & -2 & -3 & -4 \\ 0 & -3 & -6 & -9 & -12 \end{bmatrix}$$

$R_3 \rightarrow R_3 - R_2$

$R_4 \rightarrow R_4 - 3R_2$

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 & 6 \\ 0 & -1 & -2 & -3 & -4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{\rho(A) = 2}$$